

Computer Architecture

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Keep in mind there are *two* PDFs available (of which this is the latter):

1. a PDF of examinable material used as lecture slides, and
2. a PDF of non-examinable, extra material:
 - ▶ the associated notes page may be pre-populated with extra, written explanation of material covered in lecture(s), plus
 - ▶ anything with a “grey’ed out” header/footer represents extra material which is useful and/or interesting but out of scope (and hence not covered).

Notes:

Notes:

- **Agenda:** introduce the topic [4, Part 1] of

finite automata $\equiv$ **Finite State Machines (FSMs)**

via

1. an “in theory”, i.e., concept-oriented perspective, and
2. an “in practice”, i.e., perspective, spanning
 - 2.1 general application, and
 - 2.2 specific implementation in sequential logic design.

Notes:

Part 1: in theory (1)

Definition

An **alphabet** is a non-empty set of **symbols**.

Definition

A **string** X with respect to some alphabet Σ is a sequence, of finite length, whose elements are members of Σ , i.e.,

$$X = \langle X_0, X_1, \dots, X_{n-1} \rangle$$

for some n such that $X_i \in \Sigma$ for $0 \leq i < n$; if n is zero, we term X the **empty string** and denote it ϵ . It can be useful, and is common to write elements in human-readable form termed a **string literal**: this basically means writing them from right-to-left without any associated notation (e.g., brackets or commas).

Definition

A **language** Λ is a set of strings.

Notes:

Definition

A (formal) **grammar** is a tuple

$$G = (N, n, T, P)$$

including

1. N , a finite set of **non-terminal symbols** that includes a **start symbol** $n \in N$,
2. T , a finite set of **terminal symbols** (which is disjoint from N), and
3. P , a finite set of **production rules** each of the form

$$P_i : (N \cup T)^* \rightarrow (N \cup T)^*.$$

Notes:

Part 1: in theory (3)

- **Concept:** **Finite State Machines (FSMs)** are a model of **computation**.

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Part 1: in theory (3)

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 - ▶ An FSM is an (idealised) computer C , which, at a given point in time, is in one of some finite set of states.

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Part 1: in theory (3)

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 1. C accepts (or recognises) the input string
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 - ▶ Once the input is exhausted, C halts: depending on the state it halts in, we say either
 1. C accepts (or recognises) the input string
 2. C rejects the input string
 - ▶ For a language Λ of all possible input strings C could accept, we say
$$C \text{ accepts (or recognises) } \Lambda \equiv \Lambda \text{ is the language of } C$$
and use Λ to classify C ...

Notes:

Part 1: in theory (4)

Definition

	less powerful more powerful				
Machine	Combinatorial logic	Finite automaton	Push-down automaton	Linear-bounded automaton	Turing machine
Memory		0 stacks	1 stacks	2 stacks	2 stacks
Language		regular	context free	context sensitive	recursively enumerable
Grammar		regular ($X \rightarrow x$ or $X \rightarrow xY$)	context free ($X \rightarrow \gamma$)	context sensitive ($\alpha X \beta \rightarrow \alpha \gamma \beta$)	unrestricted ($\alpha \rightarrow \beta$)
Chomsky-Schützenberger hierarchy		type-3	type-2	type-1	type-0

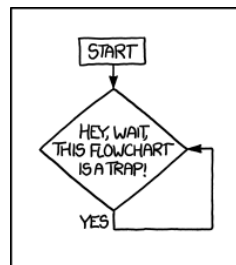
Notes:

- In the description of grammars, the idea is (read from left-to-right) there are progressively less and less restrictions: X and Y both represent non-terminal symbols, x represents a terminal symbol, and α , β and γ are strings of either terminal or non-terminal symbols, all with respect to some alphabet and grammar. So, the far right-hand case has rules that are totally unrestricted, for example, whereas the far left-hand case has rules that must be of a particular form.

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Part 1: in theory (5)



Notes:

<https://xkcd.com/1195>

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Part 1: in theory (6)

Definition

A (deterministic) **Finite State Machine (FSM)** is a tuple

$$C = (S, s, A, \Sigma, \Gamma, \delta, \omega)$$

including

1. S , a finite set of **states** that includes a **start state** $s \in S$,
2. $A \subseteq S$, a finite set of **accepting states**,
3. an **input alphabet** Σ and an **output alphabet** Γ ,
4. a **transition function**

$$\delta : S \times \Sigma \rightarrow S$$

and

5. an **output function**

$$\omega : S \rightarrow \Gamma$$

in the case of a **Moore FSM**, or

$$\omega : S \times \Sigma \rightarrow \Gamma$$

in the case of a **Mealy FSM**,

noting an **empty** input denoted ϵ allows a transition that can *always* occur.

Notes:

- Even if the state space is *huge*, it is still finite: to cope, δ can a **partial function** meaning it needn't be defined for all inputs.

Part 1: in theory (7)

- **Problem:** design an FSM that decides whether a binary sequence X has an odd number of 1 elements in it.

Notes:

Part 1: in theory (7)

► Solution:

Algorithm (tabular)			Algorithm (diagram)																	
<table><tr><td></td><td colspan="2">δ</td></tr><tr><td>Q</td><td>Q'</td><td></td></tr><tr><td></td><td>$X_i = 0$</td><td>$X_i = 1$</td></tr><tr><td>S_{even}</td><td>S_{even}</td><td>S_{odd}</td></tr><tr><td>S_{odd}</td><td>S_{odd}</td><td>S_{even}</td></tr></table>				δ		Q	Q'			$X_i = 0$	$X_i = 1$	S_{even}	S_{even}	S_{odd}	S_{odd}	S_{odd}	S_{even}	<pre>graph LR; start((start)) --> S_even((S_even)); S_even -- "X_i = 0" --> S_even; S_even -- "X_i = 1" --> S_odd((S_odd)); S_odd -- "X_i = 1" --> S_even; S_odd -- "X_i = 0" --> S_odd; style S_odd fill:#fff,stroke:#333,stroke-width:1px</pre>		
	δ																			
Q	Q'																			
	$X_i = 0$	$X_i = 1$																		
S_{even}	S_{even}	S_{odd}																		
S_{odd}	S_{odd}	S_{even}																		

where, e.g.,

- for the input string $X = \langle 1, 0, 1, 1 \rangle$ the transitions are

$\leadsto S_{even} \xrightarrow{X_0=1} S_{odd} \xrightarrow{X_1=0} S_{odd} \xrightarrow{X_2=1} S_{even} \xrightarrow{X_3=1} S_{odd}$

so the input is accepted (i.e., has an odd number of 1 elements).

- for the input string $X = \langle 0, 1, 1, 0 \rangle$ the transitions are

$\leadsto S_{even} \xrightarrow{X_0=0} S_{even} \xrightarrow{X_1=1} S_{odd} \xrightarrow{X_2=1} S_{even} \xrightarrow{X_3=0} S_{even}$

so the input is rejected (i.e., has an even number of 1 elements).

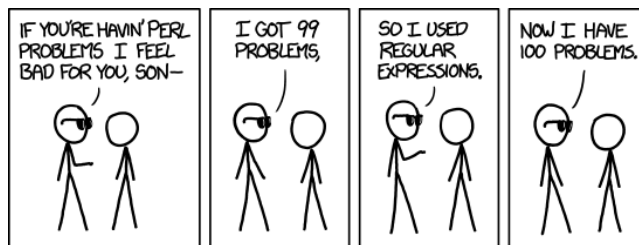
Notes:

Part 2.1: in practice, application (1)

Example #1: regular expressions + `grep` $\leadsto$ FSMs as recognisers

► Context:

► -ve perspective:



► +ve perspective: we could say that

arithmetic expression	evaluate $\leadsto$	number
regular expression	evaluate $\leadsto$	language

so a regular expression (or regex) can be used as

- a pattern used to describe or generate a language, *or*
- a pattern used to identify (i.e., match) members of a language.

Notes:

Definition

We say X is a **regular expression** if it is

1. a symbol in the alphabet, i.e., $\{x\}$ for $x \in \Sigma$,
2. the union of regular expressions X and Y such that
$$X \cup Y = \{x \mid x \in X \vee x \in Y\},$$
3. the concatenation of regular expressions X and Y such that
$$X \parallel Y = \{\langle x, y \rangle \mid x \in X \wedge y \in Y\},$$
or
4. the **Kleene star** of regular expression X such that
$$X^* = \{\langle x_0, x_1, \dots, x_{n-1} \rangle \mid n \geq 0, x_i \in X\}.$$

allowing for various short-hands, e.g.,

x	$\equiv$	$\{x\}$
xy	$\equiv$	$\{x\} \parallel \{y\}$
X^+	$\equiv$	$X \parallel X^*$

Notes:

► **Example** [4, Example 1.53]: if $\Sigma = \{0, 1\}$, then

$$0^*10^* \equiv \left\{ s \mid \begin{array}{l} s \text{ is a string containing} \\ \text{a single } 1 \end{array} \right\}$$

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- The example implementation in the `grep` example is a little simplistic, so the behaviour (and hence output) will differ versus use of `grep` itself. The technical reason for this is that we have ignored (or disallowed) backtracking: if the FSM accepts a ‘ 1 ’ followed by something other than ‘ 0 ’ then it fails, but obviously this ignores the possibility that there might be *another* ‘ 1 ’ later which *is* followed by ‘ 0 ’. So the description might be better written as “ s is a line read from `stdin`, where a) there is at least one ‘ 1 ’ character, and b) the first ‘ 1 ’ character is followed by at least one ‘ 0 ’ character”. We could try to capture that using a more complex transition function, but then the example becomes overly complex: although the details is important in a general sense, the underlying idea is more important at this point. In *even* more detail, the difference here relates to a difference between so-called Deterministic Finite Automatons (DFAs) and Non-deterministic Finite Automatons (NFAs): we are dealing with and assume the former, whereas `grep` makes use of the latter (see, e.g., [4, Section 1.2]).

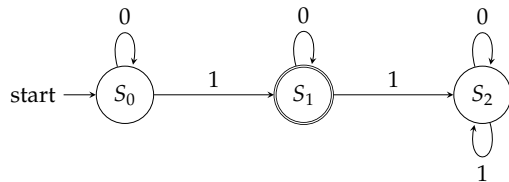
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► **Example** [4, Example 1.53]: if $\Sigma = \{0, 1\}$, then

$$\Sigma^*001\Sigma^* \equiv \left\{ s \mid \begin{array}{l} s \text{ is a string containing} \\ 001 \text{ as a sub-string} \end{array} \right\}$$

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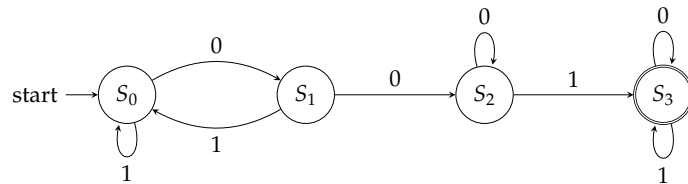
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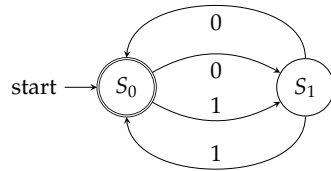
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► **Example** [4, Example 1.53]: if $\Sigma = \{0, 1\}$, then

$$1^*(01^+)^* \equiv \left\{ s \mid \begin{array}{l} s \text{ is a string in which} \\ \text{every 0 is followed by at least one 1} \end{array} \right\}$$

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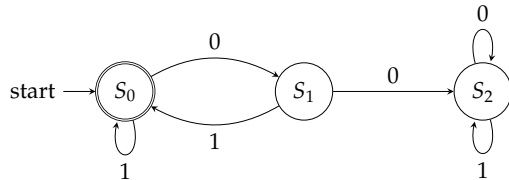
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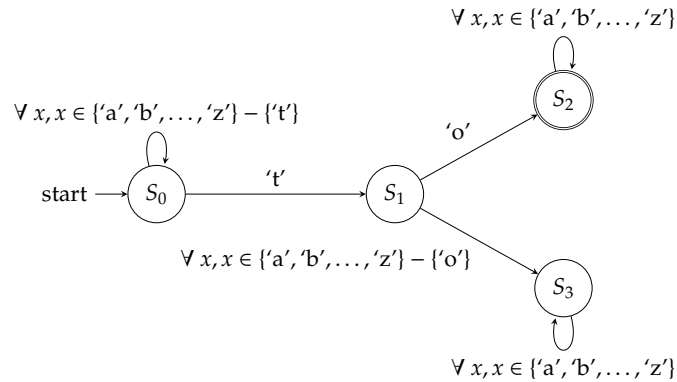
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```

1 forall lines X read from stdin do
3   Q ← s
4   for i = 0 upto n - 1 do
5     | Q ← δ(Q, Xi)
6   end
7   if Q ∈ A then
8     | print line X to stdout
10  end
11 end
  
```

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which can be (sort of, with a significant caveat) realised using

```
1 void grep() {
2   char X[ 1024 ];
3
4   while( NULL != fgets( X, 1024, stdin ) ) {
5     int n = strlen( X ), Q = start;
6
7     if( X[ n - 1 ] == '\n' ) {
8       X[ n - 1 ] = '\0'; n--;
9     }
10
11    for( int i = 0; i < n; i++ ) {
12      Q = delta[ Q ][ X[ i ] ];
13    }
14
15    if( accept[ Q ] ) {
16      fprintf( stdout, "%s\n", X );
17    }
18  }
19 }
```

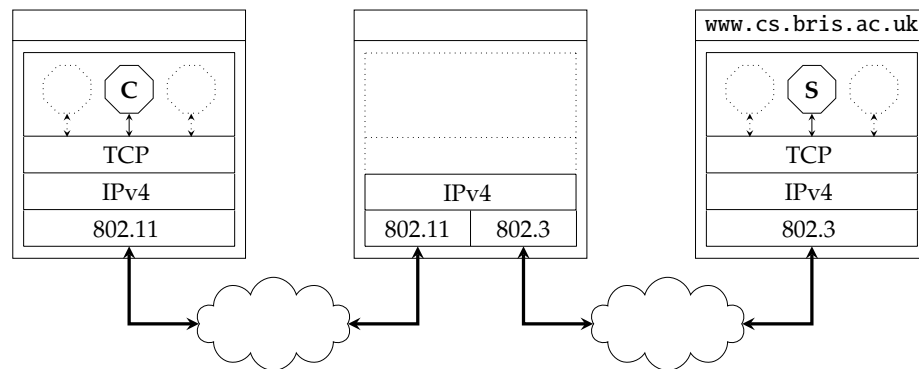
Notes:

- The example implementation in the `grep` example is a little simplistic, so the behaviour (and hence output) will differ versus use of `grep` itself. The technical reason for this is that we have ignored (or disallowed) backtracking; if the FSM accepts a 't' followed by something other than 'o' then it fails, but obviously this ignores the possibility that there might be *another* 't' later which is followed by 'o'. So the description might be better written as "s is a line read from `stdin`, where a) there is at least one 't' character, and b) the first 't' character is followed by at least one 'o' character". We could try to capture that using a more complex transition function, but then the example becomes overly complex: although the details is important in a general sense, the underlying idea is more important at this point. In *even* more detail, the difference here relates to a difference between so-called Deterministic Finite Automatons (DFAs) and Non-deterministic Finite Automatons (NFAs): we are dealing with and assume the former, whereas `grep` makes use of the latter (see, e.g., [4, Section 1.2]).

Part 2.1: in practice, application (4)

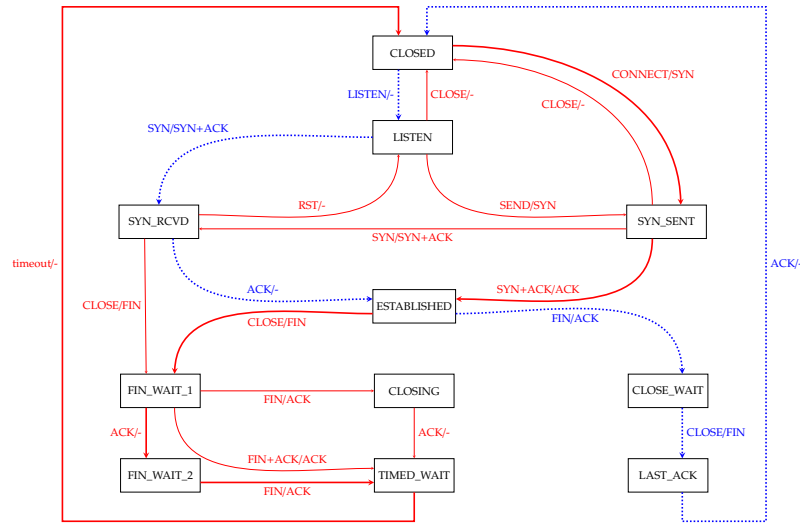
Example #2: networked communication via TCP $\leadsto$ FSMs as controllers

► **Context:**



Notes:

► Example:



Notes:

► Context:



Algorithm

```
1 reset the game state
2 while  $\neg$  game over do
3   read control pad (e.g., check if button pressed)
4   update game state (e.g., move player)
5   produce graphics and/or sound
6 end
```

Notes:

► Context:



Algorithm

```

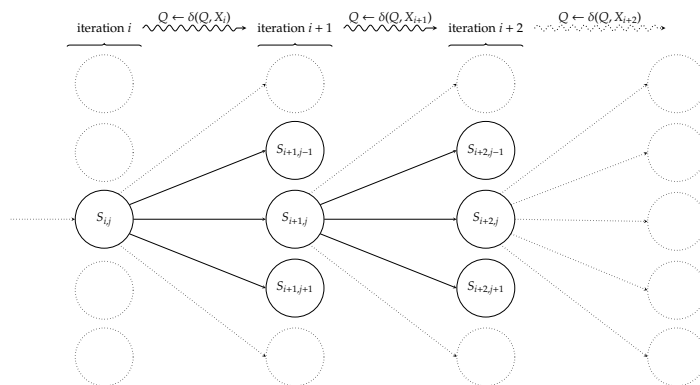
1  $Q \leftarrow s$ 
2 while  $Q \notin A$  do
3    $X_i \leftarrow \text{control pad}$ 
4    $Q \leftarrow \delta(Q, X_i)$ 
5    $\{\text{graphics, sound}\} \leftarrow \omega(Q)$ 
6 end
    
```

Notes:

► Example:

iterations of game loop $\leadsto$ game tree
 $\simeq$ state space

i.e.,



which is most obvious with respect to turn-based games (e.g., chess).

Notes:

► Recall:

Definition

A (deterministic) **Finite State Machine (FSM)** is a tuple

$$C = (S, s, A, \Sigma, \Gamma, \delta, \omega)$$

including

1. S , a finite set of **states** that includes a **start state** $s \in S$,
2. $A \subseteq S$, a finite set of **accepting states**,
3. an **input alphabet** Σ and an **output alphabet** Γ ,
4. a **transition function**

$$\delta : S \times \Sigma \rightarrow S$$

and

5. an **output function**

$$\omega : S \rightarrow \Gamma$$

in the case of a **Moore FSM**, or

$$\omega : S \times \Sigma \rightarrow \Gamma$$

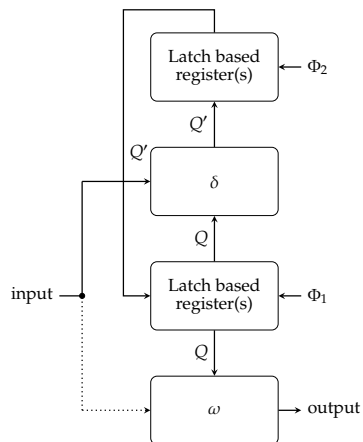
in the case of a **Mealy FSM**,

noting an **empty** input denoted ϵ allows a transition that can *always* occur.

Notes:

► Concept:

Algorithm (latch version)



► Note that

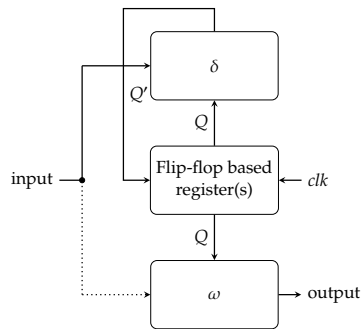
1. the state is retained in a register (i.e., a group of latches, resp. flip-flops),
2. δ and ω are simply combinatorial logic,
3. within the current clock cycle
 - ω computes the output from the current state and input, and
 - δ computes the next state from the current state and input,
4. the next state is latched by an appropriate feature (i.e., level, resp. edge) in the clock

i.e., it's a *computer* we can *build*!

Notes:

► Concept:

Algorithm (flip-flop version)



► Note that

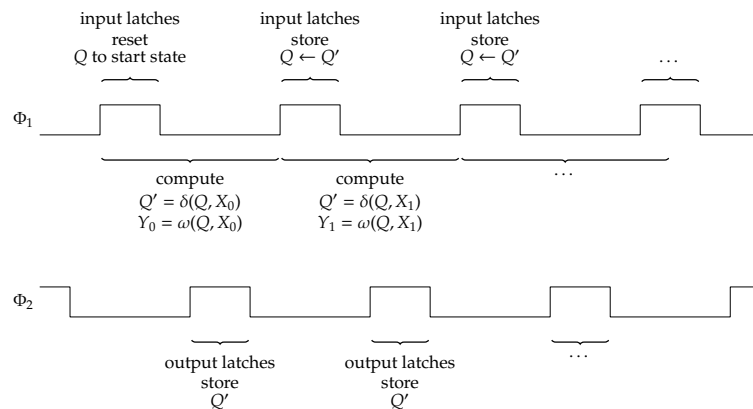
1. the state is retained in a register (i.e., a group of latches, resp. flip-flops),
2. δ and ω are simply combinatorial logic,
3. within the current clock cycle
 - ω computes the output from the current state and input, and
 - δ computes the next state from the current state and input,
4. the next state is latched by an appropriate feature (i.e., level, resp. edge) in the clock

i.e., it's a *computer* we can *build*!

Notes:

► Concept:

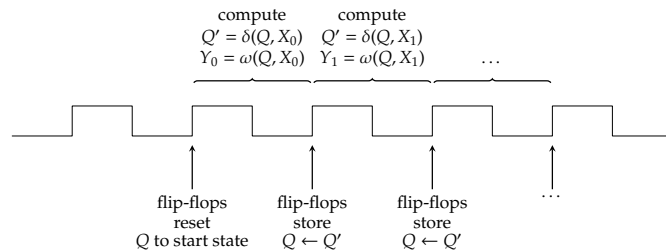
Example (latch version: input $X = \langle X_0, X_1, \dots \rangle$, output $Y = \langle Y_0, Y_1, \dots \rangle$)



Notes:

► Concept:

Example (flip-flop version: input $X = \langle X_0, X_1, \dots \rangle$, output $Y = \langle Y_0, Y_1, \dots \rangle$)

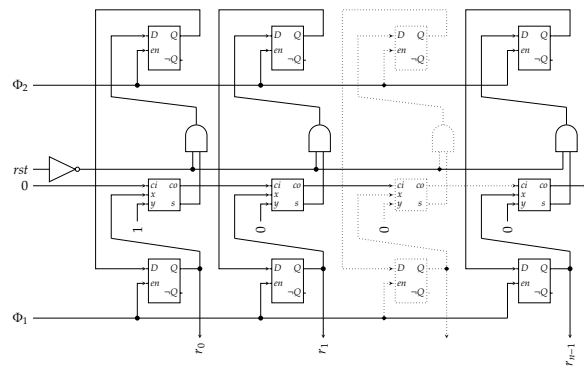


Notes:

Part 2.2: in practice, implementation (6)
Design framework

► Concept: this *should* sound familiar, because from

Circuit (latch version)



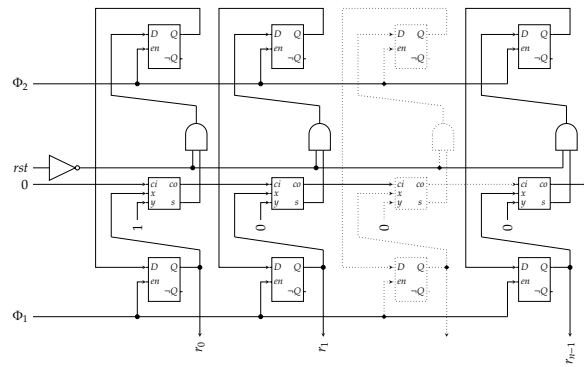
it *now* becomes clear that

- 2^n states, labelled S_0 through S_{2^n-1} ; state S_i represented as (unsigned) n -bit integer i ,
- the start state is $s = S_0$ and there are no accepting states (so $A = \emptyset$),

Notes:

- **Concept:** this *should* sound familiar, because from

Circuit (latch version)



it *now* becomes clear that

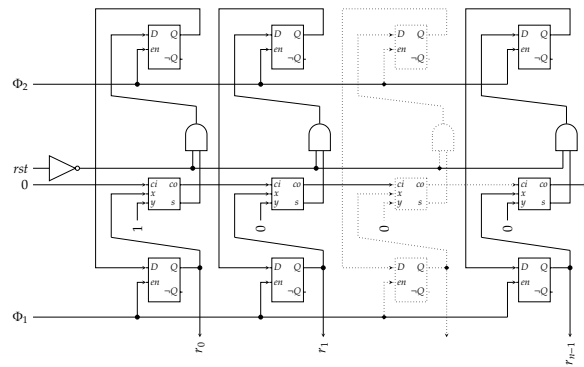
- the δ function is

$$Q' \leftarrow \delta(Q, rst) = \begin{cases} Q + 1 \pmod{2^n} & \text{if } rst = 0 \\ 0 & \text{if } rst = 1 \end{cases}$$

Notes:

- **Concept:** this *should* sound familiar, because from

Circuit (latch version)



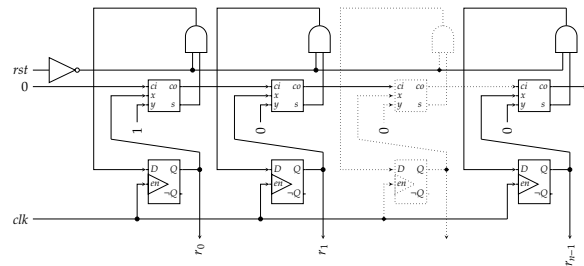
it *now* becomes clear that

- the ω function is $r \leftarrow \omega(Q) = Q$.

Notes:

- **Concept:** this *should* sound familiar, because from

Circuit (flip-flop version)



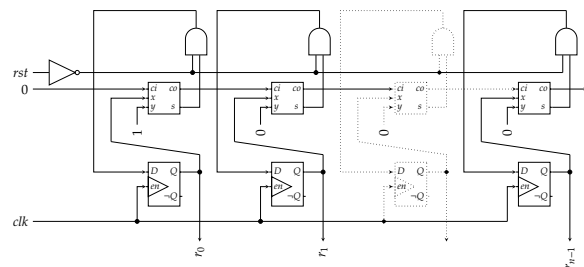
it *now* becomes clear that

- 2^n states, labelled S_0 through S_{2^n-1} ; state S_i represented as (unsigned) n -bit integer i ,
- the start state is $s = S_0$ and there are no accepting states (so $A = \emptyset$),

Notes:

- **Concept:** this *should* sound familiar, because from

Circuit (flip-flop version)



it *now* becomes clear that

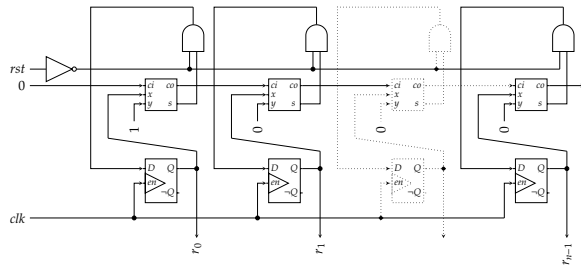
- the δ function is

$$Q' \leftarrow \delta(Q, rst) = \begin{cases} Q + 1 & (\text{mod } 2^n) \\ 0 & \end{cases} \begin{cases} \text{if } rst = 0 \\ \text{if } rst = 1 \end{cases}$$

Notes:

- **Concept:** this *should* sound familiar, because from

Circuit (flip-flop version)



it *now* becomes clear that

- the ω function is $r \leftarrow \omega(Q) = Q$.

Notes:

- **Concept:** to solve a concrete problem, we follow a (fairly) standard sequence of steps

Algorithm

1. Count the number of states required, and give each state an abstract label.
2. Describe the state transition and output functions using a tabular or diagrammatic approach.
3. Perform state assignment, i.e., decide how concrete values will represent the abstract labels, allocating appropriate register(s) to hold the state.
4. Express the functions δ and ω as (optimised) Boolean expressions, i.e., combinatorial logic.
5. Place the registers and combinatorial logic into the framework.

noting that it's common to

- include a **reset** input that (re)initialises the FSM into the start state,
- replace the accepting state(s) with an **idle** or **error** state since "halting" doesn't make sense in hardware, and
- use the FSM to control an associated data-path using the outputs, rather than (necessarily) solve some problem outright.

Notes:

► **Concept:** we can optimise the state representation based on use of it, e.g.,

1. a **binary encoding** represents the i -th of n states as a $(\lceil \log_2(n) \rceil)$ -bit unsigned integer i , e.g.,

S_0	$\mapsto$	$\langle 0, 0, 0 \rangle$
S_1	$\mapsto$	$\langle 1, 0, 0 \rangle$
S_2	$\mapsto$	$\langle 0, 1, 0 \rangle$
S_3	$\mapsto$	$\langle 1, 1, 0 \rangle$
S_4	$\mapsto$	$\langle 0, 0, 1 \rangle$
S_5	$\mapsto$	$\langle 1, 0, 1 \rangle$

2. a **one-hot encoding** represents the i -th of n states as a sequence X such that $X_i = 1$ and $X_j = 0$ for $j \neq i$, e.g.,

S_0	$\mapsto$	$\langle 1, 0, 0, 0, 0, 0 \rangle$
S_1	$\mapsto$	$\langle 0, 1, 0, 0, 0, 0 \rangle$
S_2	$\mapsto$	$\langle 0, 0, 1, 0, 0, 0 \rangle$
S_3	$\mapsto$	$\langle 0, 0, 0, 1, 0, 0 \rangle$
S_4	$\mapsto$	$\langle 0, 0, 0, 0, 1, 0 \rangle$
S_5	$\mapsto$	$\langle 0, 0, 0, 0, 0, 1 \rangle$

noting that we have a larger state (i.e., n bits instead of $\lceil \log_2(n) \rceil$), *but*

- transition between states is easier, *and*
- switching behaviour (and hence power consumption) is reduced.

Notes:

Part 2.2: in practice, implementation (10)

Example #1: a modulo 6 ascending counter

► **Problem:** design an FSM that acts as a cyclic counter modulo $n = 6$ (versus 2^n).

Notes:

- If $n = 6$ for example, then we want a component whose output r steps through values

$0, 1, 2, 3, 4, 5, 0, 1, \dots,$

with the modular reduction representing control behaviour (versus the uncontrolled counter that was cyclic by default).

Part 2.2: in practice, implementation (10)

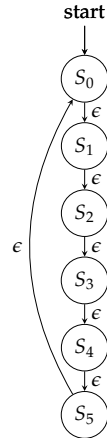
Example #1: a modulo 6 ascending counter

► Solution:

Algorithm (tabular)

	δ	ω
Q	Q'	r
S_0	S_1	0
S_1	S_2	1
S_2	S_3	2
S_3	S_4	3
S_4	S_5	4
S_5	S_0	5

Algorithm (diagram)



Notes:

- If $n = 6$ for example, then we want a component whose output r steps through values

$0, 1, 2, 3, 4, 5, 0, 1, \dots,$

with the modular reduction representing control behaviour (versus the uncontrolled counter that was cyclic by default).

Part 2.2: in practice, implementation (10)

Example #1: a modulo 6 ascending counter

► Solution:

- there are 6 abstract labels

$S_0 \mapsto 0$
 $S_1 \mapsto 1$
 $S_2 \mapsto 2$
 $S_3 \mapsto 3$
 $S_4 \mapsto 4$
 $S_5 \mapsto 5$

we can represent using 6 concrete values, e.g.,

$S_0 \mapsto \langle 0, 0, 0 \rangle \equiv 000_{(2)}$
 $S_1 \mapsto \langle 1, 0, 0 \rangle \equiv 001_{(2)}$
 $S_2 \mapsto \langle 0, 1, 0 \rangle \equiv 010_{(2)}$
 $S_3 \mapsto \langle 1, 1, 0 \rangle \equiv 011_{(2)}$
 $S_4 \mapsto \langle 0, 0, 1 \rangle \equiv 100_{(2)}$
 $S_5 \mapsto \langle 1, 0, 1 \rangle \equiv 101_{(2)}$

- since $2^3 = 8 > 6$, we can capture each of

1. $Q = \langle Q_0, Q_1, Q_2 \rangle \equiv$ the current state
2. $Q' = \langle Q'_0, Q'_1, Q'_2 \rangle \equiv$ the next state

in a 3-bit register (i.e., via 3 latches or flip-flops).

Notes:

- If $n = 6$ for example, then we want a component whose output r steps through values

$0, 1, 2, 3, 4, 5, 0, 1, \dots,$

with the modular reduction representing control behaviour (versus the uncontrolled counter that was cyclic by default).

Part 2.2: in practice, implementation (10)

Example #1: a modulo 6 ascending counter

► Solution:

- rewriting the abstract labels yields the following concrete truth table

			δ			ω		
Q_2	Q_1	Q_0	Q'_2	Q'_1	Q'_0	r_2	r_1	r_0
0	0	0	0	0	1	0	0	0
0	0	1	0	1	0	0	0	1
0	1	0	0	1	1	0	1	0
0	1	1	1	0	0	0	1	1
1	0	0	1	0	1	1	0	0
1	0	1	0	0	0	1	0	1
1	1	0	?	?	?	?	?	?
1	1	1	?	?	?	?	?	?

- note that our state assignment means $r = Q$, such that

$$r = \omega(Q) = Q$$

is basically just the identity function (so we'll ignore it).

Notes:

- If $n = 6$ for example, then we want a component whose output r steps through values

$$0, 1, 2, 3, 4, 5, 0, 1, \dots,$$

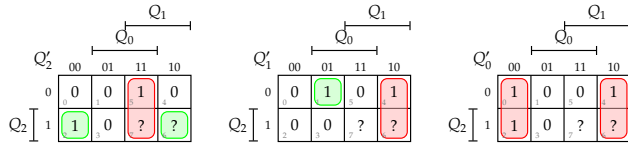
with the modular reduction representing control behaviour (versus the uncontrolled counter that was cyclic by default).

Part 2.2: in practice, implementation (10)

Example #1: a modulo 6 ascending counter

► Solution:

- the truth table can be translated into



- doing so yields the following Boolean expressions for δ :

$$Q'_2 = (\quad \quad \quad Q_1 \quad \wedge \quad Q_0 \quad) \vee (\quad Q_2 \quad \wedge \quad \neg Q_0 \quad)$$

$$Q'_1 = (\neg Q_2 \quad \wedge \quad \neg Q_1 \quad \wedge \quad Q_0 \quad) \vee (\quad \quad \quad Q_1 \quad \wedge \quad \neg Q_0 \quad)$$

$$Q'_0 = (\quad \quad \quad \neg Q_0 \quad)$$

Notes:

- If $n = 6$ for example, then we want a component whose output r steps through values

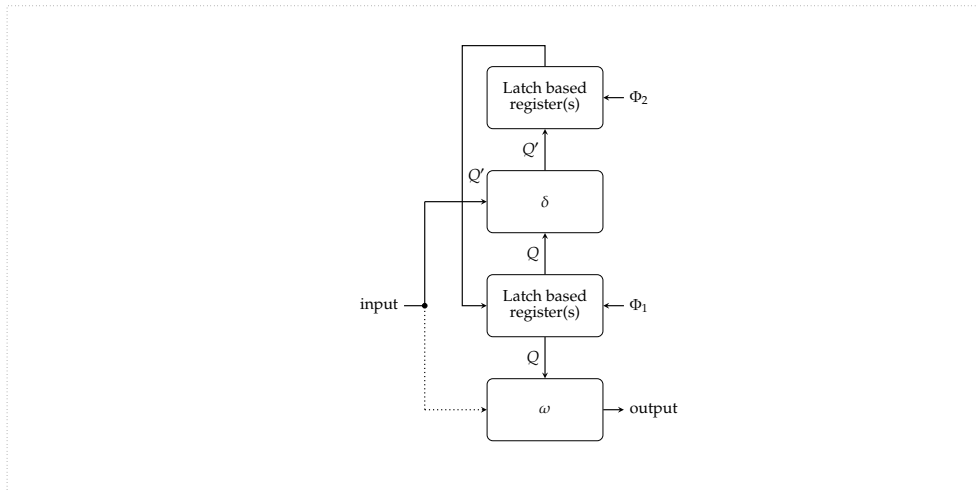
$$0, 1, 2, 3, 4, 5, 0, 1, \dots,$$

with the modular reduction representing control behaviour (versus the uncontrolled counter that was cyclic by default).

Part 2.2: in practice, implementation (10)

Example #1: a modulo 6 ascending counter

► Solution:



Notes:

- If $n = 6$ for example, then we want a component whose output r steps through values

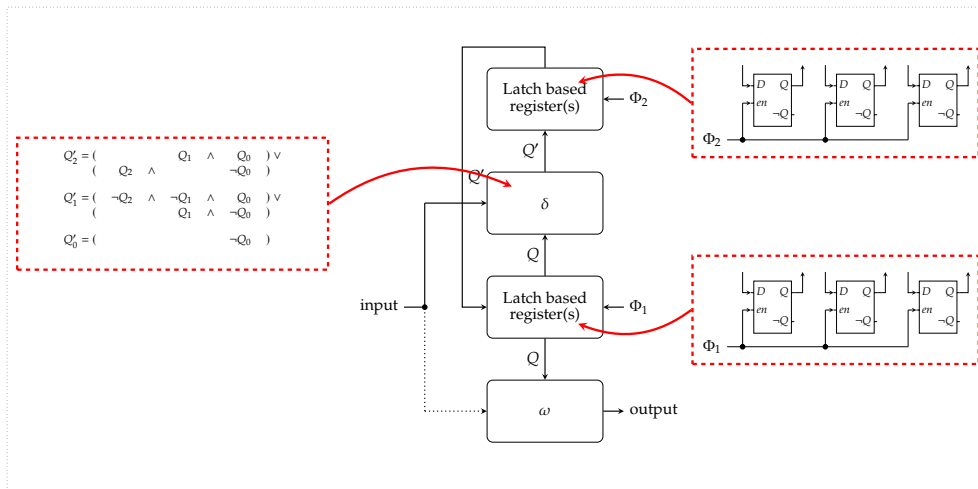
$0, 1, 2, 3, 4, 5, 0, 1, \dots,$

with the modular reduction representing control behaviour (versus the uncontrolled counter that was cyclic by default).

Part 2.2: in practice, implementation (10)

Example #1: a modulo 6 ascending counter

► Solution:



Notes:

- If $n = 6$ for example, then we want a component whose output r steps through values

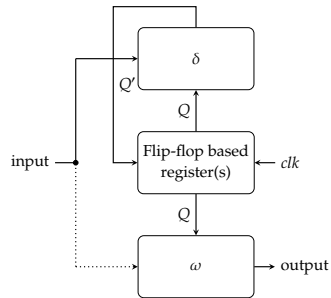
$0, 1, 2, 3, 4, 5, 0, 1, \dots,$

with the modular reduction representing control behaviour (versus the uncontrolled counter that was cyclic by default).

Part 2.2: in practice, implementation (10)

Example #1: a modulo 6 ascending counter

► Solution:



Notes:

- If $n = 6$ for example, then we want a component whose output r steps through values

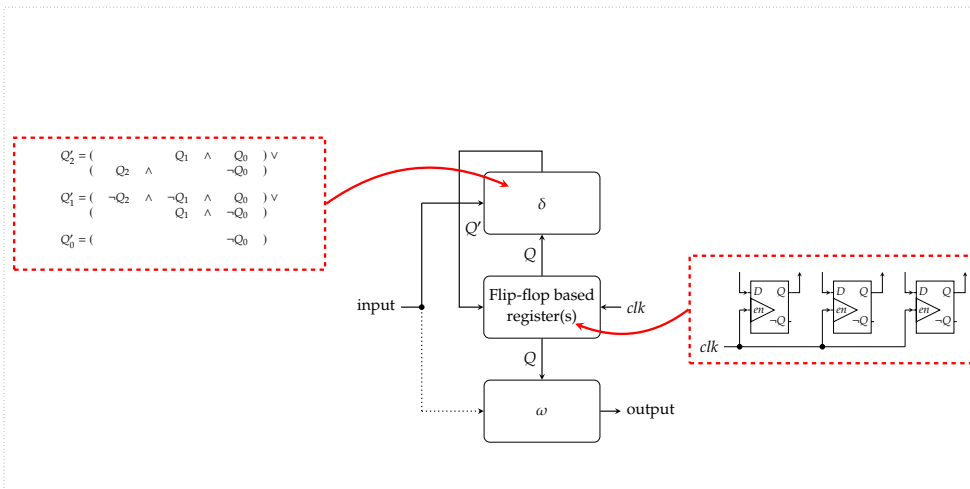
$0, 1, 2, 3, 4, 5, 0, 1, \dots,$

with the modular reduction representing control behaviour (versus the uncontrolled counter that was cyclic by default).

Part 2.2: in practice, implementation (10)

Example #1: a modulo 6 ascending counter

► Solution:



Notes:

- If $n = 6$ for example, then we want a component whose output r steps through values

$0, 1, 2, 3, 4, 5, 0, 1, \dots,$

with the modular reduction representing control behaviour (versus the uncontrolled counter that was cyclic by default).

Part 2.2: in practice, implementation (11)

Example #2: a modulo 6 ascending *or* descending counter, with cycle alert

► Problem: design an FSM that

1. acts as a cyclic counter modulo $n = 6$ (versus 2^n),
2. has an input d which selects between increment and decrement, and
3. has an output f which signals when a cycle occurs.

Notes:

- If $n = 6$, for example, then
 - if $d = 0$
 - the output r steps through values 0, 1, 2, 3, 4, 5, 0, 1, ...
 - the output $f = 1$ iff. $r = 5$
 - if $d = 1$
 - the output r steps through values 0, 5, 4, 3, 2, 1, 0, 5, ...
 - the output $f = 1$ iff. $r = 0$.

Part 2.2: in practice, implementation (11)

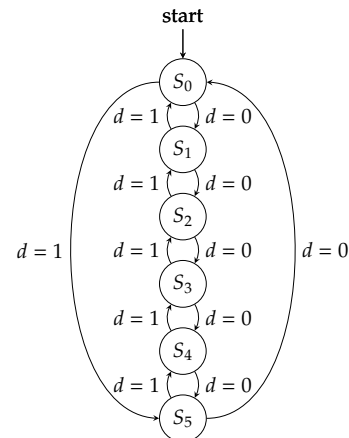
Example #2: a modulo 6 ascending *or* descending counter, with cycle alert

► Solution:

Algorithm (tabular)

Q	δ		r	ω	
	$d = 0$	$d = 1$		$d = 0$	$d = 1$
S_0	S_1	S_5	0	0	1
S_1	S_2	S_0	1	0	0
S_2	S_3	S_1	2	0	0
S_3	S_4	S_2	3	0	0
S_4	S_5	S_3	4	0	0
S_5	S_0	S_4	5	1	0

Algorithm (diagram)



Notes:

- If $n = 6$, for example, then
 - if $d = 0$
 - the output r steps through values 0, 1, 2, 3, 4, 5, 0, 1, ...
 - the output $f = 1$ iff. $r = 5$
 - if $d = 1$
 - the output r steps through values 0, 5, 4, 3, 2, 1, 0, 5, ...
 - the output $f = 1$ iff. $r = 0$.

Part 2.2: in practice, implementation (11)

Example #2: a modulo 6 ascending *or* descending counter, with cycle alert

► Solution:

- there are 6 abstract labels

$$\begin{array}{ll} S_0 & \mapsto 0 \\ S_1 & \mapsto 1 \\ S_2 & \mapsto 2 \\ S_3 & \mapsto 3 \\ S_4 & \mapsto 4 \\ S_5 & \mapsto 5 \end{array}$$

we can represent using 6 concrete values, e.g.,

$$\begin{array}{llll} S_0 & \mapsto & \langle 0, 0, 0 \rangle & \equiv 000_{(2)} \\ S_1 & \mapsto & \langle 1, 0, 0 \rangle & \equiv 001_{(2)} \\ S_2 & \mapsto & \langle 0, 1, 0 \rangle & \equiv 010_{(2)} \\ S_3 & \mapsto & \langle 1, 1, 0 \rangle & \equiv 011_{(2)} \\ S_4 & \mapsto & \langle 0, 0, 1 \rangle & \equiv 100_{(2)} \\ S_5 & \mapsto & \langle 1, 0, 1 \rangle & \equiv 101_{(2)} \end{array}$$

- since $2^3 = 8 > 6$, we can capture each of

1. $Q = \langle Q_0, Q_1, Q_2 \rangle \equiv$ the current state
2. $Q' = \langle Q'_0, Q'_1, Q'_2 \rangle \equiv$ the next state

in a 3-bit register (i.e., via 3 latches or flip-flops).

Notes:

- If $n = 6$, for example, then
 - if $d = 0$
 - the output r steps through values 0, 1, 2, 3, 4, 5, 0, 1, ...
 - the output $f = 1$ iff. $r = 5$
 - if $d = 1$
 - the output r steps through values 0, 5, 4, 3, 2, 1, 0, 5, ...
 - the output $f = 1$ iff. $r = 0$.

Part 2.2: in practice, implementation (11)

Example #2: a modulo 6 ascending *or* descending counter, with cycle alert

► Solution:

- rewriting the abstract labels yields the following concrete truth table

				δ			ω			
d	Q_2	Q_1	Q_0	Q'_2	Q'_1	Q'_0	r_2	r_1	r_0	f
0	0	0	0	0	0	1	0	0	0	0
0	0	0	1	0	1	0	0	0	1	0
0	0	1	0	0	1	1	0	1	0	0
0	0	1	1	1	0	0	0	1	1	0
0	1	0	0	1	0	1	1	0	0	0
0	1	0	1	0	0	0	1	0	1	1
0	1	1	0	?	?	?	?	?	?	?
0	1	1	1	?	?	?	?	?	?	?
1	0	0	0	1	0	1	0	0	0	1
1	0	0	1	0	0	0	0	0	1	0
1	0	1	0	0	0	1	0	1	0	0
1	0	1	1	0	1	0	0	1	1	0
1	1	0	0	0	1	1	1	0	0	0
1	1	0	1	1	0	0	1	0	1	0
1	1	1	0	?	?	?	?	?	?	?
1	1	1	1	?	?	?	?	?	?	?

Notes:

- If $n = 6$, for example, then
 - if $d = 0$
 - the output r steps through values 0, 1, 2, 3, 4, 5, 0, 1, ...
 - the output $f = 1$ iff. $r = 5$
 - if $d = 1$
 - the output r steps through values 0, 5, 4, 3, 2, 1, 0, 5, ...
 - the output $f = 1$ iff. $r = 0$.

Part 2.2: in practice, implementation (11)

Example #2: a modulo 6 ascending *or* decending counter, with cycle alert

► Solution:

- the truth table can be translated into

Three truth tables for Q_2' , Q_1' , and Q_0' are shown, each with columns Q_2 (00, 01, 11, 10) and Q_1 (00, 01, 11, 10). The rows represent Q_0 (00, 01, 11, 10). The tables show values for Q_2' , Q_1' , and Q_0' based on the inputs Q_2 , Q_1 , and Q_0 . Some cells are highlighted in green, red, or blue to indicate specific logic conditions.

- doing so yields the following Boolean expressions for δ :

$$Q_2' = (\neg d \quad \quad \quad \wedge \quad Q_1 \quad \wedge \quad Q_0) \vee \\ (\neg d \quad \wedge \quad Q_2 \quad \quad \quad \wedge \quad \neg Q_0) \vee \\ (\quad d \quad \wedge \quad Q_2 \quad \quad \quad \wedge \quad Q_0) \vee \\ (\quad d \quad \wedge \quad \neg Q_2 \quad \wedge \quad \neg Q_1 \quad \wedge \quad \neg Q_0)$$

$$Q_1' = (\neg d \quad \wedge \quad \neg Q_2 \quad \wedge \quad \neg Q_1 \quad \wedge \quad Q_0) \vee \\ (\neg d \quad \quad \quad \wedge \quad Q_1 \quad \wedge \quad \neg Q_0) \vee \\ (\quad d \quad \wedge \quad Q_2 \quad \quad \quad \wedge \quad \neg Q_0) \vee \\ (\quad d \quad \quad \quad \wedge \quad Q_1 \quad \wedge \quad Q_0)$$

$$Q_0' = (\quad \quad \quad \neg Q_0)$$

Notes:

- If $n = 6$, for example, then
 - if $d = 0$
 - the output r steps through values 0, 1, 2, 3, 4, 5, 0, 1, ...
 - the output $f = 1$ iff. $r = 5$
 - if $d = 1$
 - the output r steps through values 0, 5, 4, 3, 2, 1, 0, 5, ...
 - the output $f = 1$ iff. $r = 0$.

Part 2.2: in practice, implementation (11)

Example #2: a modulo 6 ascending *or* decending counter, with cycle alert

► Solution:

- the truth table can be translated into

A truth table for f is shown, with columns Q_2 (00, 01, 11, 10) and Q_1 (00, 01, 11, 10). The rows represent Q_0 (00, 01, 11, 10). The table shows values for f based on the inputs Q_2 , Q_1 , and Q_0 . Some cells are highlighted in red or green to indicate specific logic conditions.

- doing so yields the following Boolean expressions for ω :

$$f = (\neg d \quad \wedge \quad Q_2 \quad \quad \quad \wedge \quad Q_0) \vee \\ (\quad d \quad \wedge \quad \neg Q_2 \quad \wedge \quad \neg Q_1 \quad \wedge \quad \neg Q_0)$$

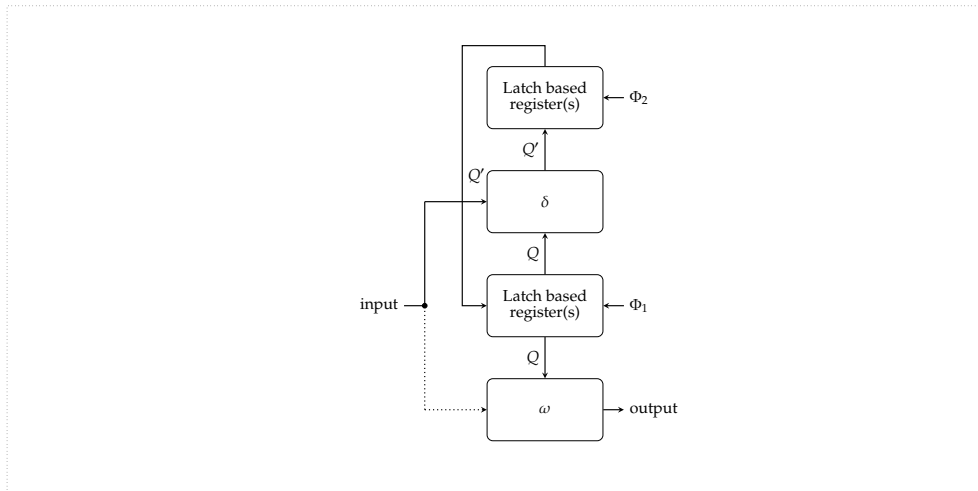
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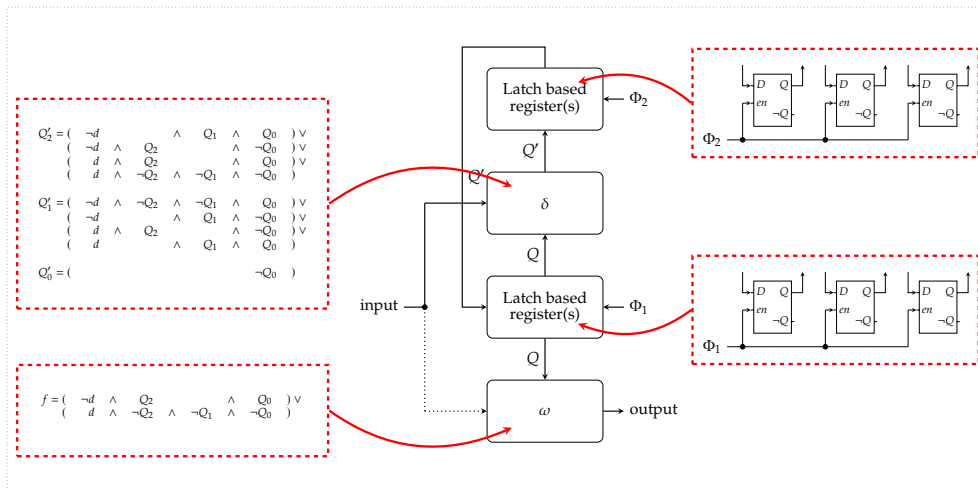
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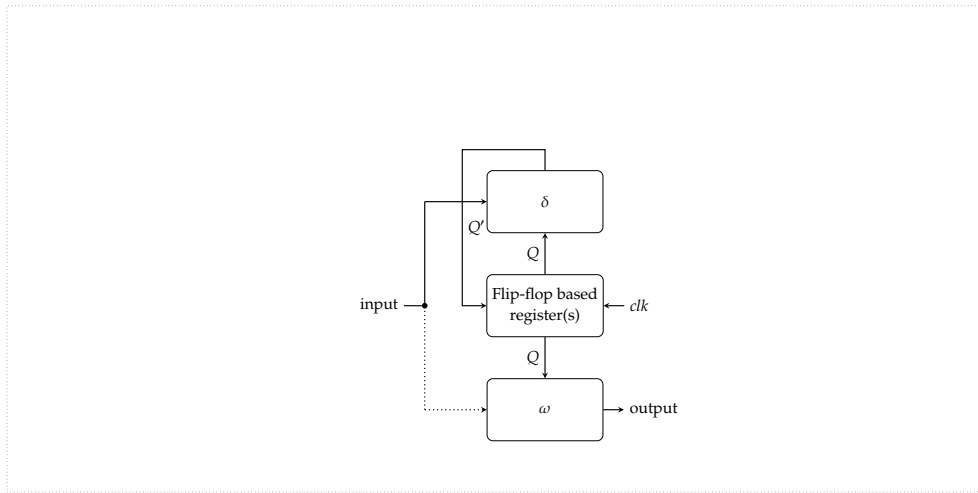
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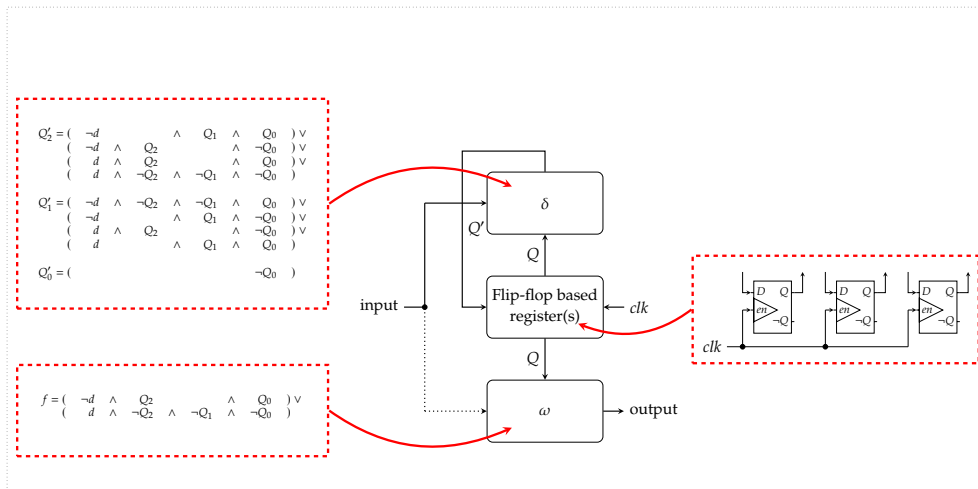
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Part 2.2: in practice, implementation (12)

Example #3: a loop counter

► Problem: design an FSM that

- replicates the behaviour of a controlled loop counter, e.g., `i` within a C-style for loop such as

```
for( int i = m; i < n; i++ ) {  
    ...  
}
```

- has an interface that allows signalling for

the start of iteration $\equiv$ so $i = m$
the end of iteration $\equiv$ when $i = n$

focused wlog. on 4-bit values of i , m , and n .

Notes:

Part 2.2: in practice, implementation (13)

Example #3: a loop counter

► Design:

- given a user C_1 of some component C_2 , how does
 - C_2 know when to start computation (e.g., when any input x is available), and
 - C_1 know when computation has finished (e.g., when any output $r = f(x)$ is available).
- we could implement an the interface which
 - uses a shared clock signal to synchronise events,
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 - ...

Notes:

- A shared clock might *seem* the ideal option, but there are (at least) two scenarios where it doesn't work out so well:
 - When C_2 and C_1 are physically separate: distribution of a synchronised clock will be significantly hard(er). and
 - When the number of steps C_2 takes is variable: even if C_2 and C_1 are synchronised, the latter still needs some way to tell when the former has completed a given computation.
- The diagrammatic example can be explained as follows:
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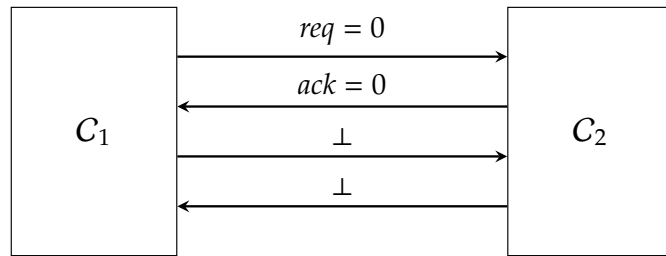
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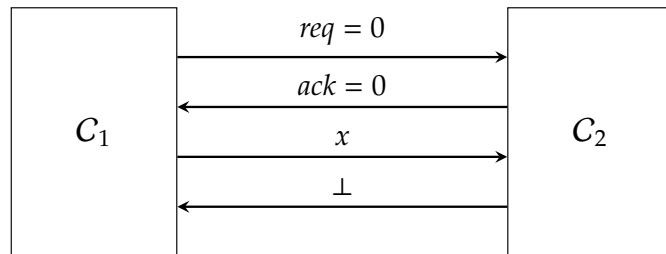
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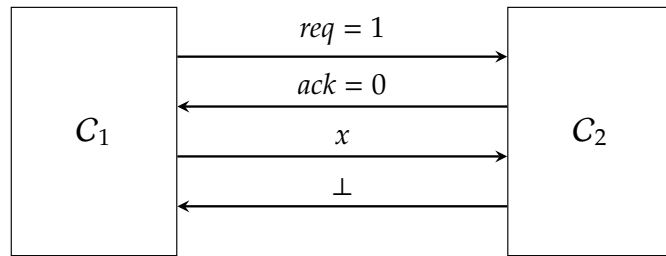
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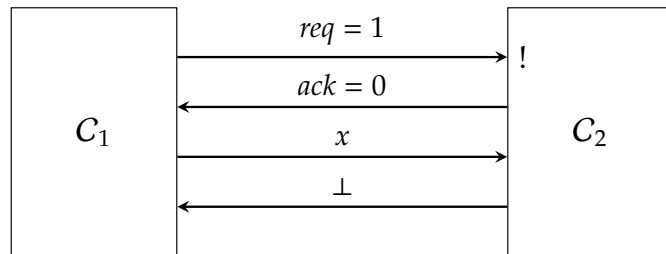
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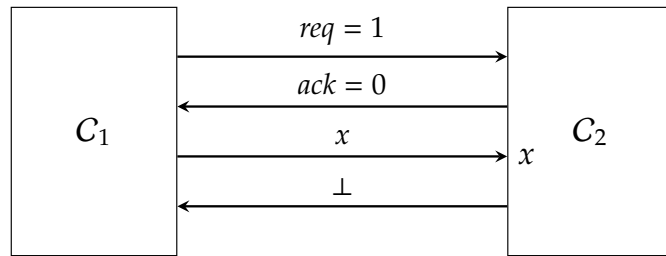
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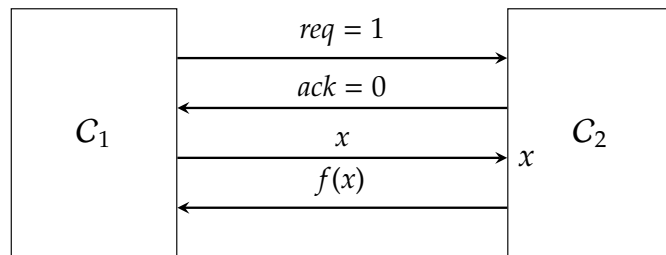
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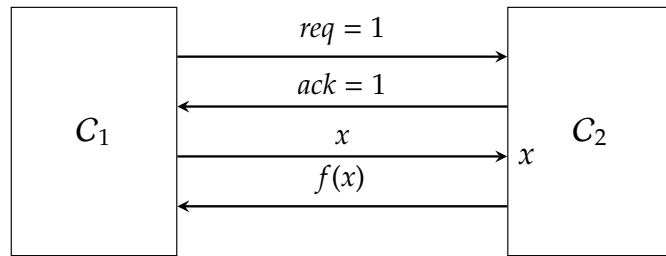
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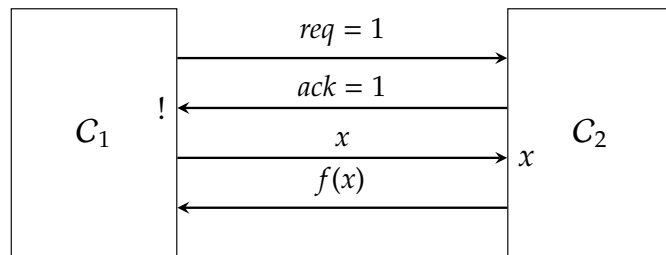
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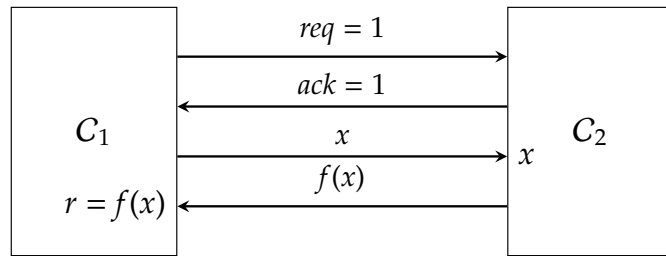
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Part 2.2: in practice, implementation (13)

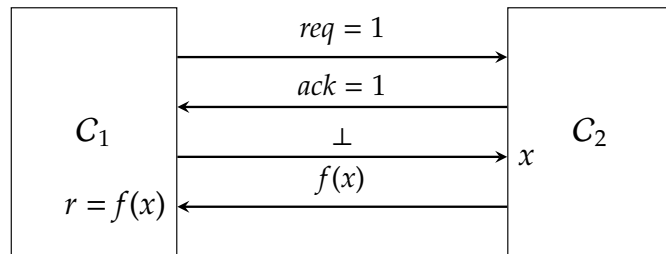
Example #3: a loop counter

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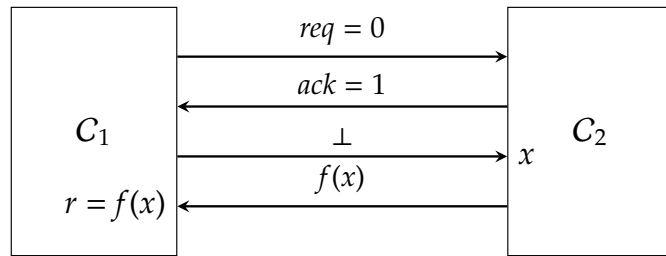
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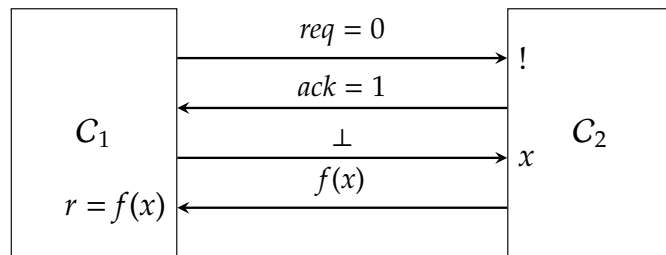
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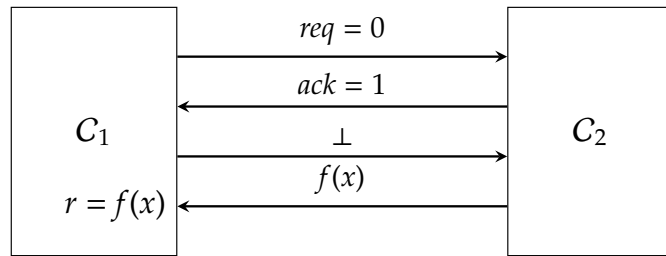
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Algorithm



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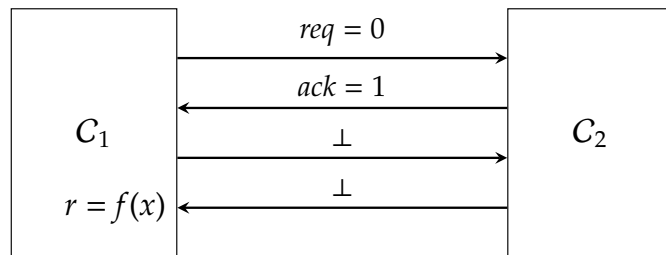
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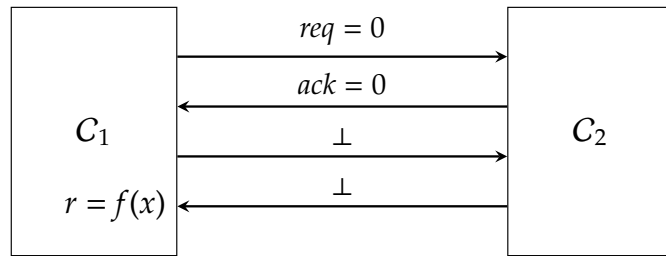
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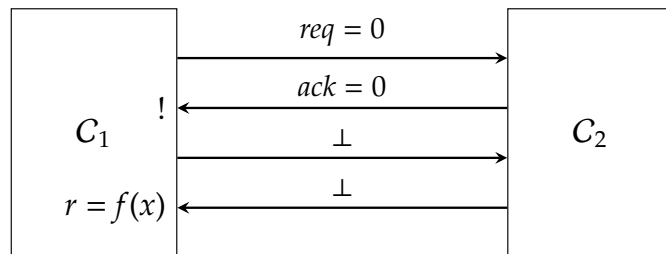
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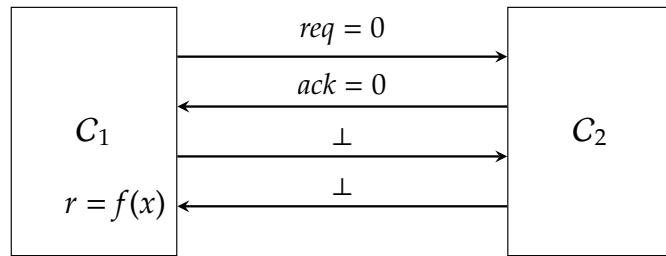
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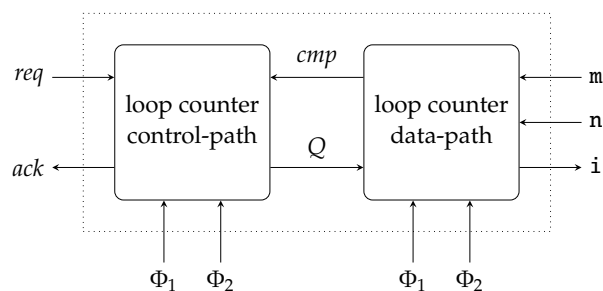
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Part 2.2: in practice, implementation (14)

Example #3: a loop counter

► Design:

Circuit (latch version)



i.e., the design is *itself* the combination of

- a **data-path**, of computational and/or storage components, and
- a **control-path**, that tells components in the data-path what to do and when to do it, with the latter more overtly realised using an FSM.

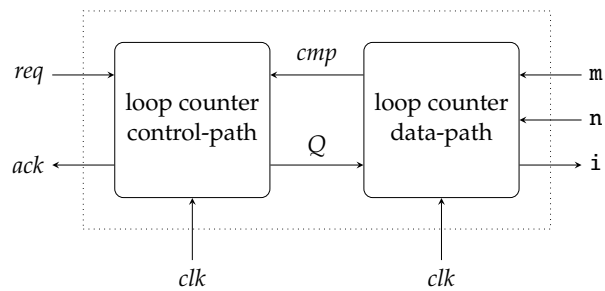
Notes:

Part 2.2: in practice, implementation (14)

Example #3: a loop counter

► Design:

Circuit (flip-flop version)



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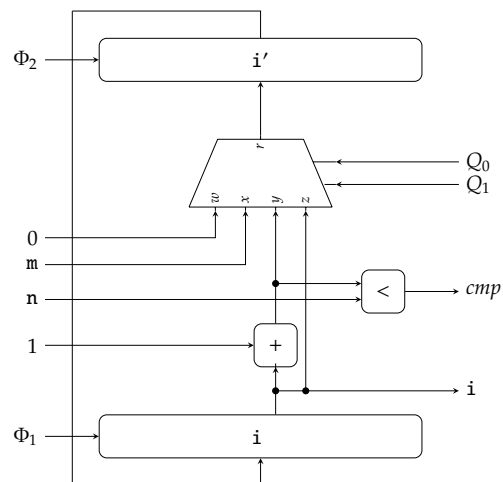
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Part 2.2: in practice, implementation (15)

Example #3: a loop counter

► Solution: the data-path.

Circuit (latch version)



Notes:

- The general structure here is the same as the previous, uncontrolled counter example: for example there is 1) there is still a register to store the current counter state, 2) there is still have an adder, but 3) there is *also* now a multiplexer to decide between several options for the next state.
- Some of the inputs (e.g., Q) and outputs (e.g., cmp) shown need to be considered in combination with the control-path. For example, it should now be clear that
 - if $Q = (0, 0) \mapsto S_{init}$, then the multiplexer updates the output register with 0,
 - if $Q = (1, 0) \mapsto S_{init}$ then the multiplexer updates the output register with m , i.e., the initial counter value,
 - if $Q = (0, 1) \mapsto S_{step}$ then the multiplexer updates the output register with $i + 1$, i.e., the incremented counter value produced by the adder,
 - if $Q = (1, 1) \mapsto S_{done}$ then the multiplexer updates the output register with i , i.e., the current counter value.

Likewise, it should be clear that

$$cmp = \begin{cases} 0 & \text{if } i < n \\ 1 & \text{if } i \geq n \end{cases}$$

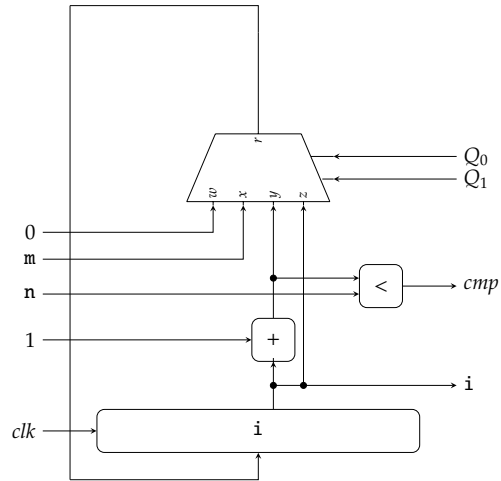
which controls the transition from S_{step} into S_{done} .

Part 2.2: in practice, implementation (15)

Example #3: a loop counter

► **Solution:** the data-path.

Circuit (flip-flop version)



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- Some of the inputs (e.g., Q) and outputs (e.g., cmp) shown need to be considered in combination with the control-path. For example, it should now be clear that
 - if $Q = \langle 0, 0 \rangle \mapsto S_{wait}$, then the multiplexer updates the output register with 0,
 - if $Q = \langle 1, 0 \rangle \mapsto S_{init}$ then the multiplexer updates the output register with m , i.e., the initial counter value,
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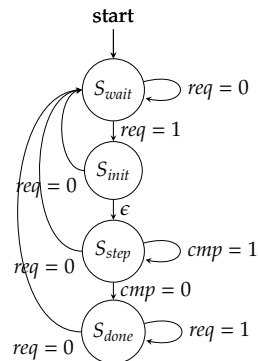
Example #3: a loop counter

► **Solution:** the control-path.

Algorithm (tabular)

	Q	δ		ω	
		Q'	ack	$cmp = 0$	$cmp = 1$
$req = 0$	S_{wait}	S_{wait}	S_{wait}	0	0
	S_{init}	S_{wait}	S_{wait}	0	0
	S_{step}	S_{wait}	S_{wait}	0	0
	S_{done}	S_{wait}	S_{wait}	1	1
$req = 1$	S_{wait}	S_{init}	S_{init}	0	0
	S_{init}	S_{step}	S_{step}	0	0
	S_{step}	S_{done}	S_{step}	0	0
	S_{done}	S_{done}	S_{done}	1	1

Algorithm (diagram)



i.e.,

- in S_{wait} it waits for $req = 1$,
- in S_{init} it uses any input to initialise itself (e.g., setting the initial loop counter value),
- in S_{step} it performs an iteration of the loop, and
- in S_{done} it waits for $req = 0$ while setting $ack = 1$.

Notes:

Part 2.2: in practice, implementation (16)

Example #3: a loop counter

► Solution: the control-path.

- there are 4 abstract labels

$$\begin{array}{ll} S_{wait} & \mapsto 0 \\ S_{init} & \mapsto 1 \\ S_{step} & \mapsto 2 \\ S_{done} & \mapsto 3 \end{array}$$

we can represent using 4 concrete values, e.g.,

$$\begin{array}{llll} S_{wait} & \mapsto & \langle 0, 0 \rangle & \equiv 00_{(2)} \\ S_{init} & \mapsto & \langle 1, 0 \rangle & \equiv 01_{(2)} \\ S_{step} & \mapsto & \langle 0, 1 \rangle & \equiv 10_{(2)} \\ S_{done} & \mapsto & \langle 1, 1 \rangle & \equiv 11_{(2)} \end{array}$$

- since $2^2 = 4$, we can capture each of

1. $Q = \langle Q_0, Q_1 \rangle \equiv$ the current state
2. $Q' = \langle Q'_0, Q'_1 \rangle \equiv$ the next state

in a 2-bit register (i.e., via 2 latches or flip-flops).

Notes:

Part 2.2: in practice, implementation (16)

Example #3: a loop counter

► Solution: the control-path.

- rewriting the abstract labels yields the following concrete truth table

<i>req</i>	<i>cmp</i>			δ		ω
		Q_1	Q_0	Q'_1	Q'_0	<i>ack</i>
0	0	0	0	0	0	0
0	0	0	1	0	0	0
0	0	1	0	0	0	0
0	0	1	1	0	0	1
0	1	0	0	0	0	0
0	1	0	1	0	0	0
0	1	1	0	0	0	0
0	1	1	1	0	0	1
1	0	0	0	0	1	0
1	0	0	1	1	0	0
1	0	1	0	1	1	0
1	0	1	1	1	1	1
1	1	0	0	0	1	0
1	1	0	1	1	0	0
1	1	1	0	1	0	0
1	1	1	1	1	1	1

Notes:

Part 2.2: in practice, implementation (16)

Example #3: a loop counter

- **Solution:** the control-path.
 - the truth table can be translated into

		Q_1			
		Q_0			
Q_1'		00	01	11	10
00	0	0	0	0	0
01	0	0	0	0	0
11	0	1	1	1	0
10	0	1	1	1	0

		Q_1			
		Q_0			
Q_0'		00	01	11	10
00	0	0	0	0	0
01	0	0	0	0	0
11	1	0	1	1	0
10	1	0	1	1	0

- doing so yields the following Boolean expressions for δ :

$$Q_1' = \left(\begin{array}{l} req \\ req \end{array} \wedge Q_1 \wedge Q_0 \right) \vee$$

$$Q_0' = \left(\begin{array}{l} req \\ req \\ req \end{array} \wedge \neg Q_1 \wedge \neg Q_0 \right) \vee \left(\begin{array}{l} req \\ req \end{array} \wedge \neg cmp \wedge Q_1 \wedge Q_0 \right) \vee$$

Notes:

Part 2.2: in practice, implementation (16)

Example #3: a loop counter

- **Solution:** the control-path.
 - the truth table can be translated into

		Q_1			
		Q_0			
ack		00	01	11	10
00	0	0	0	1	0
01	0	0	0	1	0
11	0	0	0	1	0
10	0	0	0	1	0

- doing so yields the following Boolean expressions for ω :

$$ack = Q_1 \wedge Q_0$$

Notes:

Part 2.2: in practice, implementation (17)

Example #3: a loop counter

Use-case:

- we want(ed) to implement a bit-serial multiplier, i.e.,

Algorithm	Circuit
Input: Two unsigned, n-bit, base-2 integers x and y Output: An unsigned, $2n$-bit, base-2 integer $r = y \cdot x$ <pre> 1 $r \leftarrow 0$ 2 for $i = n - 1$ downto 0 step -1 do 3 $r \leftarrow 2 \cdot r$ 4 if $y_i = 1$ then 5 $r \leftarrow r + x$ 6 end 7 end 8 return r </pre>	

- we *did* have the data-path,
- we *didn't* have the control-path.

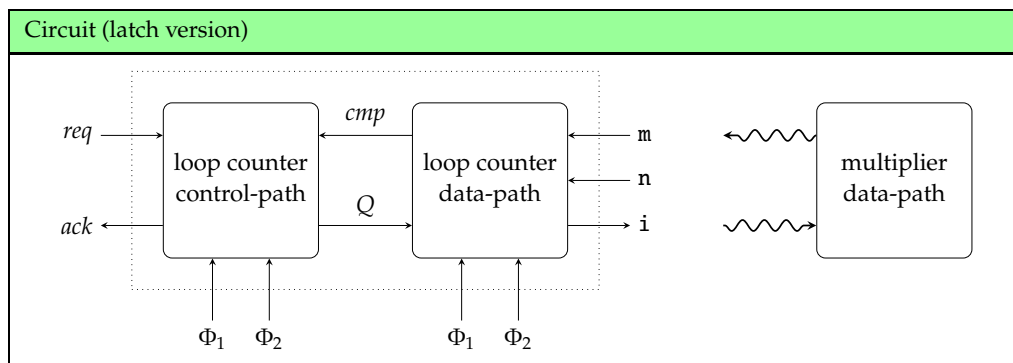
Notes:

Part 2.2: in practice, implementation (18)

Example #3: a loop counter

Use-case:

- we now have the loop counter implemented, i.e.



- the remaining challenge is integration, e.g., specifying
 - any additional data-path components required, and
 - how loop counter (the control-path) controls them
 so we end up with a bit-serial multiplier.

Notes:

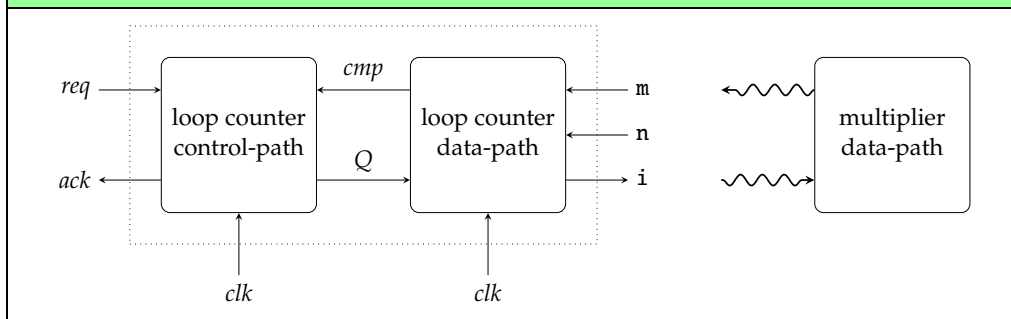
Part 2.2: in practice, implementation (18)

Example #3: a loop counter

► Use-case:

- we now have the loop counter implemented, i.e.

Circuit (flip-flop version)



- the remaining challenge is integration, e.g., specifying
 - any additional data-path components required, and
 - how loop counter (the control-path) controls them
- so we end up with a bit-serial multiplier.

Notes:

Part 2.2: in practice, implementation (19)

Example #4: a traffic light controller

- **Problem:** design an FSM that controls two sets of UK-style traffic lights:
 - the traffic lights at the intersection is between a main road and an access road,
 - they should stop cars crashing into each other, displaying
 - green on main road and red on access road, then
 - amber on main road and red on access road, then
 - red on main road and amber on access road, then
 - red on main road and green on access road, then
 - red on main road and amber on access road, then
 - amber on main road and red on access road,
 - and then cycle, and
 - they should allow use of an emergency stop button, which
 - forces red on both main and access roads while pushed, then
 - reset the system into an initial start state when released.

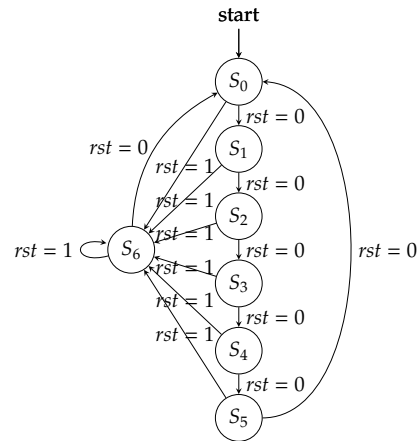
Notes:

► Solution:

Algorithm (tabular)

Q	δ		ω					
	Q'		M_g	M_d	M_r	A_g	A_d	A_r
	$rst = 0$	$rst = 1$						
S_0	S_1	S_6	1	0	0	0	0	1
S_1	S_2	S_6	0	1	0	0	0	1
S_2	S_3	S_6	0	0	1	0	1	0
S_3	S_4	S_6	0	0	1	1	0	0
S_4	S_5	S_6	0	0	1	0	1	0
S_5	S_0	S_6	0	1	0	0	0	1
S_6	S_0	S_6	0	0	1	0	0	1

Algorithm (diagram)



Notes:

► Solution:

- there are 7 abstract labels

S_0	$\mapsto$	0
S_1	$\mapsto$	1
S_2	$\mapsto$	2
S_3	$\mapsto$	3
S_4	$\mapsto$	4
S_5	$\mapsto$	5
S_6	$\mapsto$	6

we can represent using 7 concrete values, e.g.,

S_0	$\mapsto$	$\langle 0, 0, 0 \rangle$	$\equiv$	$000_{(2)}$
S_1	$\mapsto$	$\langle 1, 0, 0 \rangle$	$\equiv$	$001_{(2)}$
S_2	$\mapsto$	$\langle 0, 1, 0 \rangle$	$\equiv$	$010_{(2)}$
S_3	$\mapsto$	$\langle 1, 1, 0 \rangle$	$\equiv$	$011_{(2)}$
S_4	$\mapsto$	$\langle 0, 0, 1 \rangle$	$\equiv$	$100_{(2)}$
S_5	$\mapsto$	$\langle 1, 0, 1 \rangle$	$\equiv$	$101_{(2)}$
S_6	$\mapsto$	$\langle 0, 1, 1 \rangle$	$\equiv$	$110_{(2)}$

- since
- $2^3 = 8 > 7$
- , we can capture each of

1. $Q = \langle Q_0, Q_1, Q_2 \rangle \equiv$ the current state
2. $Q' = \langle Q'_0, Q'_1, Q'_2 \rangle \equiv$ the next state

in a 3-bit register (i.e., via 3 latches or flip-flops).

Notes:

Part 2.2: in practice, implementation (19)

Example #4: a traffic light controller

► Solution:

- rewriting the abstract labels yields the following concrete truth table

				δ			ω					
rst	Q_2	Q_1	Q_0	Q'_2	Q'_1	Q'_0	M_g	M_a	M_r	A_g	A_a	A_r
0	0	0	0	0	0	1	1	0	0	0	0	1
0	0	0	1	0	1	0	0	1	0	0	0	1
0	0	1	0	0	1	1	0	0	1	0	1	0
0	0	1	1	1	0	0	0	0	1	1	0	0
0	1	0	0	1	0	1	0	0	1	0	1	0
0	1	0	1	0	0	0	0	1	0	0	0	1
0	1	1	0	0	0	0	0	0	1	0	0	1
0	1	1	1	?	?	?	?	?	?	?	?	?
1	0	0	0	1	1	0	1	0	0	0	0	1
1	0	0	1	1	1	0	0	1	0	0	0	1
1	0	1	0	1	1	0	0	0	1	0	1	0
1	0	1	1	1	1	0	0	0	1	1	0	0
1	1	0	0	1	1	0	0	0	1	0	1	0
1	1	0	1	1	1	0	0	1	0	0	0	1
1	1	1	0	1	1	0	0	0	1	0	0	1
1	1	1	1	?	?	?	?	?	?	?	?	?

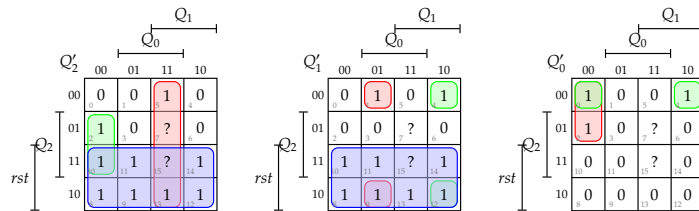
Notes:

Part 2.2: in practice, implementation (19)

Example #4: a traffic light controller

► Solution:

- the truth table can be translated into



- doing so yields the following Boolean expressions for δ :

$$Q'_2 = (rst \wedge Q_2 \wedge \neg Q_1 \wedge \neg Q_0) \vee (rst \wedge Q_1 \wedge Q_0)$$

$$Q'_1 = (rst \wedge \neg Q_2 \wedge \neg Q_1 \wedge Q_0) \vee (rst \wedge \neg Q_2 \wedge Q_1 \wedge \neg Q_0)$$

$$Q'_0 = (\neg rst \wedge \neg Q_1 \wedge \neg Q_0) \vee (\neg rst \wedge \neg Q_2 \wedge \neg Q_0)$$

Notes:

Solution:

- the truth table can be translated into

		Q_1			
		Q_0			
Q_2		00	01	11	10
	0	1	0	0	0
	1	0	0	?	0

		Q_1			
		Q_0			
Q_2		00	01	11	10
	0	0	1	0	0
	1	0	1	?	0

		Q_1			
		Q_0			
Q_2		00	01	11	10
	0	0	0	1	1
	1	1	0	?	1

		Q_1			
		Q_0			
Q_2		00	01	11	10
	0	0	0	1	0
	1	0	0	?	0

		Q_1			
		Q_0			
Q_2		00	01	11	10
	0	0	0	0	1
	1	1	0	?	0

		Q_1			
		Q_0			
Q_2		00	01	11	10
	0	1	1	0	0
	1	0	1	?	1

- doing so yields the following Boolean expressions for δ :

$$\begin{aligned}
 M_g &= (\neg Q_2 \wedge \neg Q_1 \wedge \neg Q_0) \\
 M_a &= (\neg Q_1 \wedge Q_0) \\
 M_r &= (Q_1) \vee \\
 & \quad (Q_2 \wedge \neg Q_0) \\
 A_g &= (Q_1 \wedge Q_0) \\
 A_a &= (\neg Q_2 \wedge Q_1 \wedge \neg Q_0) \vee \\
 & \quad (Q_2 \wedge \neg Q_1 \wedge \neg Q_0) \\
 A_r &= (\neg Q_2 \wedge \neg Q_1) \vee \\
 & \quad (\neg Q_1 \wedge Q_0) \vee \\
 & \quad (Q_2 \wedge Q_1)
 \end{aligned}$$

Notes:

Conclusions

Take away points:

- FSMs are abstract computational models, but we can use them to solve concrete problems, e.g.,
 - recognisers,
 - controllers,
 - ...
 - specifications*: like an algorithm, but more easily able to cater for asynchronous events.
- The “killer application” of FSMs for us is as a general-purpose way to realise controlled step-by-step forms of computation.
- Clearly more complex problem $\Rightarrow$ more complex solution, *but*
 - same framework and process (both conceptual, *and* practical),
 - same components (e.g., interface, implementation; data-path, control-path),
 so difference is (arguably) creativity re. design.

Notes:

Additional Reading

- ▶ *Wikipedia: Finite State Machine (FSM)*. URL: https://en.wikipedia.org/wiki/Finite-state_machine.
- ▶ D. Page. “Chapter 2: Basics of digital logic”. In: *A Practical Introduction to Computer Architecture*. 1st ed. Springer, 2009.
- ▶ M. Sipster. “Chapter 1: Regular languages”. In: *Introduction to the Theory of Computation*. 2nd ed. Thomson Course Technology, 2006.

Notes:

References

- [1] *Wikipedia: Finite State Machine (FSM)*. URL: https://en.wikipedia.org/wiki/Finite-state_machine (see p. 209).
- [2] D. Page. “Chapter 2: Basics of digital logic”. In: *A Practical Introduction to Computer Architecture*. 1st ed. Springer, 2009 (see p. 209).
- [3] M. Sipster. “Chapter 1: Regular languages”. In: *Introduction to the Theory of Computation*. 2nd ed. Thomson Course Technology, 2006 (see p. 209).
- [4] M. Sipster. *Introduction to the Theory of Computation*. 2nd ed. Thomson Course Technology, 2006 (see pp. 5, 35–58).

Notes: