Computer Architecture

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September 17, 2026

Keep in mind there are *two* PDFs available (of which this is the latter):

1. a PDF of examinable material used as lecture slides, and
2. a PDF of non-examinable, extra material:
   - the associated notes page may be pre-populated with extra, written explaination of material covered in lecture(s), plus
   - anything with a "grey'ed out" header/footer represents extra material which is useful and/or interesting but out of scope (and hence not covered).

Notes:

Notes:

- Context: this is a **HP-35** calculator

noting that it was

- originally released in 1972 and discontinued in 1975 with 300,000+ units sold,
- originally priced $395, i.e., about the same as a modern laptop (!),
- posthumously named an IEEE Milestone [2] in 2009.

Notes:

- Some of the limitations (or simplications) include:
  - The HP-35 could store and process 56-bit floating-point values, represented using BCD; we focus on 8-bit integer values, represented using two's-complement.
  - The HP-35 could compute a wide range of operations, including

arithmetic	:	addition, subtraction, multiplication, division
trigonometry	:	sin, arc sin, cos, arc cos, tan, arc tan
logarithms	:	$log_{10} x$, $log_e x$, e^x
other	:	$1/x$, $\sqrt{x}$, x^y, π

    We focus on a basic set arithmetic operations, namely addition, subtraction, and multiplication.

COMS10015 lecture: week #9

- Agenda: justify the claim

$$\text{FSM} + \text{arithmetic} \rightsquigarrow \text{calculator} \simeq \text{micro-processor},$$

by exploring a (limited) HP-35 implementation based on content covered so far.

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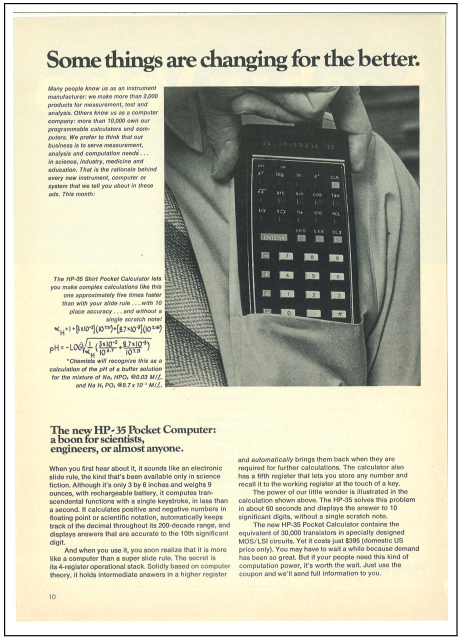
The HP-35 calculator (1)

Design



Some things are changing for the better.

Many people know us as an instrument manufacturer: we make more than 2,000 products for measurement, test and analysis. Others know us as a computer company: more than 10,000 own our programmable calculators and computers. We prefer to think that our business is to serve measurement, analysis and computation needs . . . in science, industry, medicine and education. That is the rationale behind every new instrument, computer or system that we tell you about in these ads. This month:

The HP-35 Shirt Pocket Calculator lets you make complex calculations like this one approximately five times faster than with your slide rule . . . with 10 place accuracy . . . and without a single scratch note!

$$pH = -\log\sqrt{\frac{1}{K_a}\left(\frac{[A]C_a}{[B]} + K_w\right)}$$

*Chemists will recognize this as a calculation of the pH of a buffer solution for the mixture of Na_2HPO_4 @0.03 M/L and NaH_2PO_4 @0.7 x 10^{-3} M/L.

The new HP-35 Pocket Computer: a boon for scientists, engineers, or almost anyone.

When you first hear about it, it sounds like an electronic slide rule, the kind that's been available only in science fiction. Although it's only 3 by 6 inches and weighs 9 ounces, with rechargeable battery, it computes transcendental functions with a single keystroke, in less than a second. It calculates positive and negative numbers in floating point or scientific notation, automatically keeps track of the decimal throughout its 200-decade range, and displays answers that are accurate to the 10th significant digit.

And when you use it, you soon realize that it is more like a computer than a super slide rule. The secret is its 4-register operational stack. Solidly based on computer theory, it holds intermediate answers in a higher register and automatically brings them back when they are required for further calculations. The calculator also has a fifth register that lets you store any number and recall it to the working register at the touch of a key.

The power of our little wonder is illustrated in the calculation shown above. The HP-35 solves this problem in about 60 seconds and displays the answer to 10 significant digits, without a single scratch note.

The new HP-35 Pocket Calculator contains the equivalent of 30,000 transistors in specially designed MOS/LSI circuits. Yet it costs just $395 (domestic US price only). You may have to wait a while because demand has been so great. But if your people need this kind of computation power, it's worth the wait. Just use the coupon and we'll send full information to you.

10

A faster, more efficient way to analyze drugs.

Not much can happen – neither emergency treatment for a drug overdose victim, nor prosecution of the pusher – until the drug has been positively identified. In large cities, where drug emergencies often reach into the hundreds daily, the chemist faces an enormously difficult problem, especially with traditional methods of chemical analysis. But there is a better way.

A laboratory in Charlotte (N.C.) recently sent us some powder from a confiscated pill for analysis on the new HP Gas Chromatograph/Mass Spectrometer/Computer System. Twenty minutes later, the analysis was complete: the pill contained heroin, morphine, and barbituric acid.

Fast, complete and positive, the analysis satisfied all medical and legal requirements. The HP system also takes a load off the lab's scientific staff because it can be successfully operated by technicians who have no special knowledge of mass spectroscopy or computers. The computer itself controls the operation of the spectrometer and records the mass spectrum while it makes all the necessary calculations, automatically. It can also compare the results of the analysis against a taped library of suspected components (in this case, a library of the mass spectra of 100 dangerous drugs) and automatically identify each of the sample constituents by name, positively. Where drugs are involved, that's an essential requirement. Just check the coupon for full information.

Try our solid-state lamp, free.

When Thomas Edison invented it, the incandescent lamp was one of the brightest ideas ever. It still is, if your object is illumination. But for display or indication, the new solid-state lamps (LED's) win hands down: they last at least 10 years and use only a few milliamps of current.

Because we are convinced that hands-on experience with an LED surpasses any description, we'd like to send you one of our 5082-4400 Series free. Use it even once, and we'll wager you'll never use any other kind of display lamp again.

Our LED is a gallium arsenide phosphide diode that's hard to ignore. You can see its red glow several yards away at any angle to 180°. It lights up when you connect a mere 1.6 volts to its rugged leads, draws only 2 to 10 milliamps, and includes a clip for easy panel-mounting.

Use the coupon to send for your free LED. If it turns you on, we're ready to ship quantity orders immediately.

For more information on the calculator, our systems or a free LED fill out the coupon and send to: Hewlett-Packard, 1510 Page Mill Road, Palo Alto, California 94304; Europe: P.O. Box 85, CH-1217 Meyrin 2, Geneva, Switzerland; Japan: YHP, 1-59-1, Yoyogi, Shibuya-Ku, Tokyo, 151.

HEWLETT hp PACKARD

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Notes:

- The calculator was 5.8inch long and 3.2inch wide, leading to the advertised "feature" of fitting into a standard shirt pocket.

The HP-35 calculator (2)

Design

- Concept:
  - the HP-35 uses (a variant of) **Reverse Polish Notation (RPN)**:
    - **in-fix** operators give
      $$(19 - 5) \times (1 + 2)$$
    - **pre-fix** operators (or "Polish notation") give
      $$\times\ -\ 19\ 5\ +\ 1\ 2,$$
      and finally
    - **post-fix** operators (or "reverse Polish notation") give
      $$19\ 5\ -\ 1\ 2\ +\ \times.$$
  - doing so is attractive because, for example, it
    1. is unambiguous *without* parentheses, and
    2. can be evaluated naturally using a **stack**.

Notes:

The HP-35 calculator (3)
Implementation

- External interface:

1. 'V' for $V \in \{0, 1, \ldots 9\}$
   - $X' \leftarrow 10 \cdot X + V$
2. '$\odot$' for $\odot \in \{+, -, \times\}$
   - $X' \leftarrow Y \odot X, Y' \leftarrow Z, Z' \leftarrow T, T' \leftarrow T$
3. '*CLR*' (or "clear")
   - $X' \leftarrow 0, Y' \leftarrow 0, Z' \leftarrow 0, T' \leftarrow 0$
4. '*STO*' (or "store")
   - $S' \leftarrow X$
5. '*RCL*' (or "recall")
   - $X' \leftarrow S$
6. '$\uparrow$' (or "enter")
   - $X' \leftarrow X, Y' \leftarrow X, Z' \leftarrow Y, T' \leftarrow Z$

Notes:

- As can be inferred from the instructions, the A&R unit housed registers labelled T, Z, Y and X plus S, a fifth storage register (that we often colloquially term "memory" when discussing calculators, but is *not* an SRAM or similar); the value of X is shown on the display.
- There is an interesting historical note about the HP-35 design which is relevant: the original HP-35 had a bug in the exp (or e^x) function, for example it computed

$$\exp(\ln(2.02)) = 2$$

instead of 2.02. HP had already sold 25,000 units when this bug was discovered; it (bravely) offered a refund rather than keep quiet, but in the end only ~ 5000 were returned.

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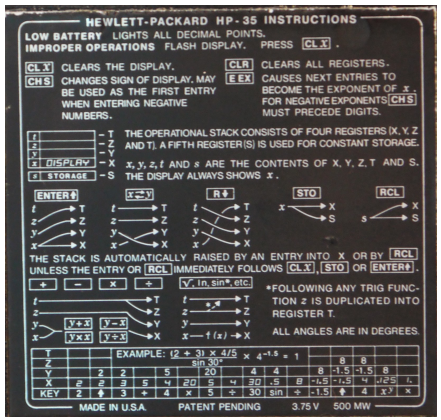

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The HP-35 calculator (4)
Implementation

- External interface:
  - consider

$$19\ 5\ -\ 1\ 2\ +\ \times$$

as evaluated using the following key presses

Key-press

Register			1	9	↑	5	−	1	↑	2	+	×
X	0	1	19	19	5	14	1	1	2	3	42
Y	0	0	0	19	19	0	14	1	1	14	0
Z	0	0	0	0	0	0	0	14	14	0	0
T	0	0	0	0	0	0	0	0	0	0	0

noting that

- ↑ signals the end of multi-digit operands,
- T, Z, Y and X are used as an evaluation stack,

  - doing so yields the result

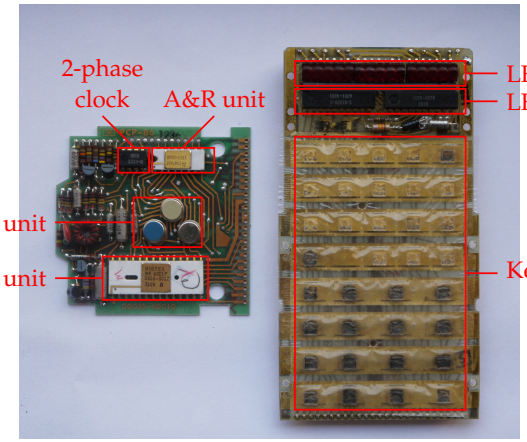
$$X = (19 - 5) \times (1 + 2) = 42$$

at the Top of Stack (ToS).

Notes:

The HP-35 calculator (5)
Implementation

- Internal implementation:



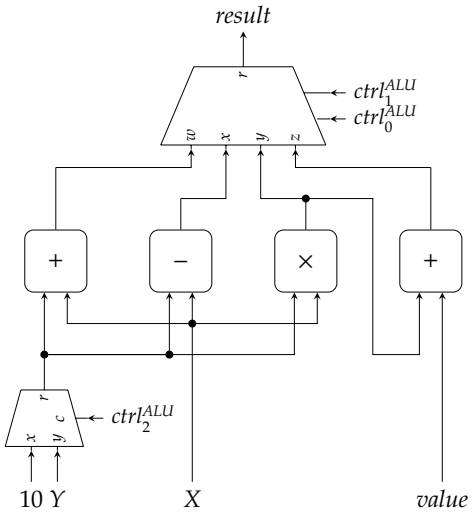
Notes:

- In more detail, the image shows
  - a 2-phase **clock generator**,
  - a **Read Only Memory (ROM)** unit,
  - an **Arithmetic and Register (A&R)** unit, and
  - a **Control and Timing (C&T)** unit

  plus a **keypad** to provide input, and an **LED-based display** to provide output.
- The LED-based characters displayed are $\frac{1}{10}$ inch high, but magnified using a spherical plastic lens: this design, e.g., reduced power consumption (versus LEDs of a larger height).
- The calculator can be powered via either 1) a mains power supply, or 2) a removable battery pack, containing three AA-sized NiCd batteries (a capacity supporting $\sim$ 3h of use).

The HP-35 calculator (6)
Implementation

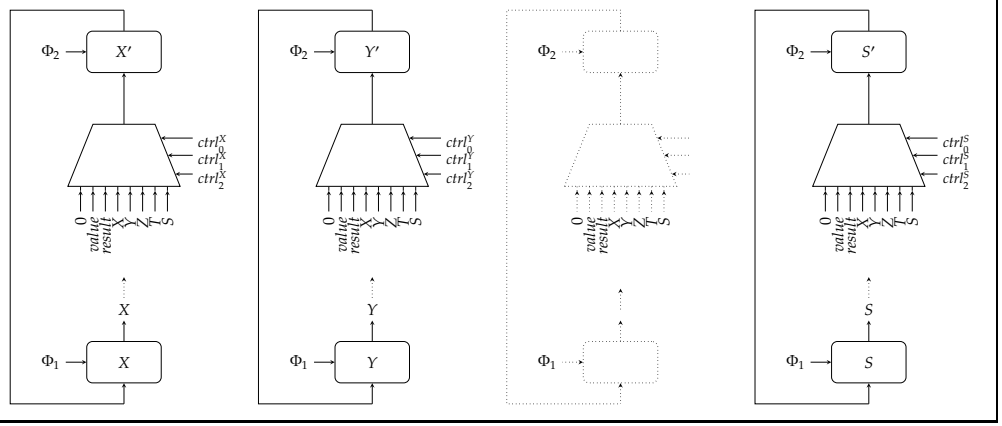
Circuit (sketch of "A" part of A&R unit)



Notes:

The HP-35 calculator (7)
Implementation

Circuit (sketch of "R" part of A&R unit)



Notes:

Demo

Notes:

- We're done: we've understood and implemented enough of a (limited) HP-35 calculator to compute

$$(19 - 5) \times (1 + 2) \quad = \quad 42,$$

but the main point is what you can do with this next:

Comparison
A **pocket calculator**
▸ has input and output **peripherals**,
▸ responds to **commands** from the user:
▸ numeric keys specifying what to perform arithmetic on, and
▸ control keys prompting arithmetic to be performed,
▸ has an **ALU** to perform arithmetic,
▸ has one or more **registers** (or accumulators),
▸ has a limited amount of **memory** (e.g., accessed via *STO* and *RCL*, or *M+* and *MR*).

Comparison
A **micro-processor**
▸ has input and output **peripherals**,
▸ executes sequences of **instructions**, i.e., **programs**:
▸ **operands** are what values to operate on, and
▸ **opcodes** determine the operation performed,
▸ has an **ALU** to perform arithmetic,
▸ has one or more **registers** (or accumulators),
▸ has (potentially) many levels and large amounts of **memory**.

Notes:

Additional Reading

- *Wikipedia: HP-35*. URL: https://en.wikipedia.org/wiki/HP-35.
- T.M. Whitney, F. Rodé, and C.C. Tung. "The "Powerful Pocketful": an Electronic Calculator Challenges the Slide Rule". In: *Hewlett-Packard Journal*. 1972, pp. 2–9.
- D.S. Cochran. "Algorithms and Accuracy in the HP-35". In: *Hewlett-Packard Journal*. 1972, pp. 10–11.
- E.T. Liljenwall. "Packaging the Pocket Calculator". In: *Hewlett-Packard Journal*. 1972, pp. 12–13.

Notes:

References

[1] *Wikipedia: HP-35*. URL: https://en.wikipedia.org/wiki/HP-35 (see p. 29).

[2] *Wikipedia: List of IEEE milestones*. URL: https://en.wikipedia.org/wiki/List_of_IEEE_milestones (see p. 5).

[3] D.S. Cochran. "Algorithms and Accuracy in the HP-35". In: *Hewlett-Packard Journal*. 1972, pp. 10–11 (see p. 29).

[4] E.T. Liljenwall. "Packaging the Pocket Calculator". In: *Hewlett-Packard Journal*. 1972, pp. 12–13 (see p. 29).

[5] T.M. Whitney, F. Rodé, and C.C. Tung. "The "Powerful Pocketful": an Electronic Calculator Challenges the Slide Rule". In: *Hewlett-Packard Journal*. 1972, pp. 2–9 (see p. 29).

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