Applied Cryptology

Daniel Page

Department of Computer Science,
University Of Bristol,
Merchant Venturers Building,
Woodland Road,
Bristol, BS8 1UB. UK.
⟨csdsp@bristol.ac.uk⟩

September 5, 2025

Keep in mind there are *two* PDFs available (of which this is the latter):

1. a PDF of examinable material used as lecture slides, and
2. a PDF of non-examinable, extra material:
   - the associated notes page may be pre-populated with extra, written explaination of material covered in lecture(s), plus
   - anything with a "grey'ed out" header/footer represents extra material which is useful and/or interesting but out of scope (and hence not covered).

Notes:

Notes:

- Agenda:
  1. a 2-part unit summary:
     - recap re. motivation, i.e., why the unit exists,
     - what did and didn't we do in the unit,
  2. drop-in slot re. coursework assignment.

Notes:

A real-world story: an attack [2] on TLS 1.2 + OpenSSL 0.9.8g (1)

Quote

The function `BN_nist_mod_384` (in `crypto/bn/bn_nist.c`) gives wrong results for some inputs.

– *Reimann [5]*

Notes:

Algorithm (NIST-P-256-Reduce, per Solinas [6, Example 3, Page 20])

Input: For $w = 32$-bit words, a 16-word integer product $z = x \cdot y$ and the modulus $p = 2^{256} - 2^{224} + 2^{192} + 2^{96} - 1$
Output: The result $r = z \pmod{p}$

1. Form the nine, 8-word intermediate variables

$$\begin{array}{lclcccccccccc}
S_0 & = & \langle & z_0, & z_1, & z_2, & z_3, & z_4, & z_5, & z_6, & z_7 & \rangle \\
S_1 & = & \langle & 0, & 0, & 0, & z_{11}, & z_{12}, & z_{13}, & z_{14}, & z_{15} & \rangle \\
S_2 & = & \langle & 0, & 0, & 0, & z_{12}, & z_{13}, & z_{14}, & z_{15}, & 0 & \rangle \\
S_3 & = & \langle & z_8, & z_9, & z_{10}, & 0, & 0, & 0, & z_{14}, & z_{15} & \rangle \\
S_4 & = & \langle & z_9, & z_{10}, & z_{11}, & z_{13}, & z_{14}, & z_{15}, & z_{13}, & z_8 & \rangle \\
S_5 & = & \langle & z_{11}, & z_{12}, & z_{13}, & 0, & 0, & 0, & z_8, & z_{10} & \rangle \\
S_6 & = & \langle & z_{12}, & z_{13}, & z_{14}, & z_{15}, & 0, & 0, & z_9, & z_{11} & \rangle \\
S_7 & = & \langle & z_{13}, & z_{14}, & z_{15}, & z_8, & z_9, & z_{10}, & 0, & z_{12} & \rangle \\
S_8 & = & \langle & z_{14}, & z_{15}, & 0, & z_9, & z_{10}, & z_{11}, & 0, & z_{13} & \rangle
\end{array}$$

2. Compute

$$r = S_0 + 2S_1 + 2S_2 + S_3 + S_4 - S_5 - S_6 - S_7 - S_8 \pmod{p}.$$

3. Return $0 \le r < p$.

Notes:

A real-world story: an attack [2] on TLS 1.2 + OpenSSL 0.9.8*g* (2)
Issue 1: arithmetic on NIST-P-{256, 384}

Algorithm (NIST-P-256-Reduce, per OpenSSL 0.9.8*g*)

Input: For $w = 32$-bit words, a 16-word integer product $z = x \cdot y$ and the modulus $p = 2^{256} - 2^{224} + 2^{192} + 2^{96} - 1$
Output: The (potentially incorrect) result $r = z \pmod{p}$

1. Form the nine, 8-word intermediate variables

$$\begin{array}{lclcccccccccc}
S_0 & = & \langle & z_0, & z_1, & z_2, & z_3, & z_4, & z_5, & z_6, & z_7 & \rangle \\
S_1 & = & \langle & 0, & 0, & 0, & z_{11}, & z_{12}, & z_{13}, & z_{14}, & z_{15} & \rangle \\
S_2 & = & \langle & 0, & 0, & 0, & z_{12}, & z_{13}, & z_{14}, & z_{15}, & 0 & \rangle \\
S_3 & = & \langle & z_8, & z_9, & z_{10}, & 0, & 0, & 0, & z_{14}, & z_{15} & \rangle \\
S_4 & = & \langle & z_9, & z_{10}, & z_{11}, & z_{13}, & z_{14}, & z_{15}, & z_{13}, & z_8 & \rangle \\
S_5 & = & \langle & z_{11}, & z_{12}, & z_{13}, & 0, & 0, & 0, & z_8, & z_{10} & \rangle \\
S_6 & = & \langle & z_{12}, & z_{13}, & z_{14}, & z_{15}, & 0, & 0, & z_9, & z_{11} & \rangle \\
S_7 & = & \langle & z_{13}, & z_{14}, & z_{15}, & z_8, & z_9, & z_{10}, & 0, & z_{12} & \rangle \\
S_8 & = & \langle & z_{14}, & z_{15}, & 0, & z_9, & z_{10}, & z_{11}, & 0, & z_{13} & \rangle
\end{array}$$

2. Compute

$$\begin{array}{lcl}
S & = & S_0 + 2S_1 + 2S_2 + S_3 + S_4 - S_5 - S_6 - S_7 - S_8 \\
  & = & t + c \cdot 2^{256}
\end{array}$$

3. Compute

$$\begin{array}{lcll}
r & = & t - c \cdot p & \pmod{2^{256}} \\
  & = & t - \mathrm{sign}(c) \cdot T[|c|] & \pmod{2^{256}}
\end{array}$$

   for pre-computed $T[i] = i \cdot p$.

4. If $r \ge p$ (resp. $r < 0$) then update $r \leftarrow r - p$ (resp. $r \leftarrow r + p$), return r.

Notes:

A real-world story: an attack [2] on TLS 1.2 + OpenSSL 0.9.8*g* (3)

Issue 1: arithmetic on NIST-P-{256, 384}

- Observation(s):
  - good: `BN_nist_mod_256` (resp. `BN_nist_mod_384`) is more efficient.
  - bad: `BN_nist_mod_256` (resp. `BN_nist_mod_384`) can produce an incorrect result, e.g.,
    1. triggered deliberately with special-form operands

$$\begin{array}{rcl} x & = & (2^{32}-1)\cdot 2^{224} + 3\cdot 2^{128} + x_0 \\ y & = & (2^{32}-1)\cdot 2^{224} + 1\cdot 2^{96} + y_0 \end{array}$$

       for random $0 \leq x_0, y_0 < 2^{32}$, *or*
    2. triggered randomly with probability $\sim 10 \cdot 2^{-29}$.

Notes:

A real-world story: an attack [2] on TLS 1.2 + OpenSSL 0.9.8*g* (4)

Issue 2: (opt-out) ephemeral-static EC-DHE

Algorithm (EC-DH(E) key agreement [7, Section 8.1][8, Section 2.1])

$\mathcal{A}$		$\mathcal{B}$
Knows $G = E(\mathbb{F}_q) = \langle G \rangle$ of order n, $pk_{\mathcal{B}}, (pk_{\mathcal{A}})^{\dagger}, (sk_{\mathcal{A}})^{\dagger}$		Knows $G = E(\mathbb{F}_q) = \langle G \rangle$ of order n, $(pk_{\mathcal{A}})^{\dagger}, pk_{\mathcal{B}}, sk_{\mathcal{B}}$
$k_{\mathcal{A}}^{(i)} \xleftarrow{\$} \{1, 2, \ldots, n-1\}$ $Q_{\mathcal{A}}^{(i)} \leftarrow [k_{\mathcal{A}}^{(i)}]G$		$k_{\mathcal{B}}^{(i)} \xleftarrow{\$} \{1, 2, \ldots, n-1\}$ $Q_{\mathcal{B}}^{(i)} \leftarrow [k_{\mathcal{B}}^{(i)}]G$
	$Q_{\mathcal{A}}^{(i)} \longrightarrow$	
	$\longleftarrow Q_{\mathcal{B}}^{(i)}$	
$R_{\mathcal{A}}^{(i)} \leftarrow [k_{\mathcal{A}}^{(i)}]Q_{\mathcal{B}}^{(i)} = [k_{\mathcal{A}}^{(i)} \cdot k_{\mathcal{B}}^{(i)}]G$		$R_{\mathcal{B}}^{(i)} \leftarrow [k_{\mathcal{B}}^{(i)}]Q_{\mathcal{A}}^{(i)} = [k_{\mathcal{B}}^{(i)} \cdot k_{\mathcal{A}}^{(i)}]G$
Use $R_{\mathcal{A}}^{(i)}$		Use $R_{\mathcal{B}}^{(i)}$

Notes:

- A high-level overview of how the above relates to OpenSSL can be found at

  https://wiki.openssl.org/index.php/Diffie_Hellman

  and

  https://wiki.openssl.org/index.php/Elliptic_Curve_Diffie_Hellman

  Note that the former explicitly warns against use of anonymous variants, offering a way to exclude them from the cipher suite list.
- It seems reasonable to say that the static-static and ephemeral-static options are confusion with respect to, e.g., the `ECDHE` cipher suite identifier (which implies ephemeral, but not which, if any party respects this).

Algorithm (EC-DH(E) key agreement [7, Section 8.1][8, Section 2.4])

$\mathcal{A}$		$\mathcal{B}$
Knows $G = E(\mathbb{F}_q) = \langle G \rangle$ of order n, $pk_{\mathcal{B}}$, $(pk_{\mathcal{A}})^{\dagger}$, $(sk_{\mathcal{A}})^{\dagger}$		Knows $G = E(\mathbb{F}_q) = \langle G \rangle$ of order n, $(pk_{\mathcal{A}})^{\dagger}$, $pk_{\mathcal{B}}$, $sk_{\mathcal{B}}$
$k_{\mathcal{A}} \xleftarrow{\$} \{1, 2, \ldots, n-1\}$ $Q_{\mathcal{A}} \leftarrow [k_{\mathcal{A}}]\, G$		$k_{\mathcal{B}} \xleftarrow{\$} \{1, 2, \ldots, n-1\}$ $Q_{\mathcal{B}} \leftarrow [k_{\mathcal{B}}]\, G$
	$\xrightarrow{Q_{\mathcal{A}}}$	
	$\xleftarrow{Q_{\mathcal{B}}}$	
$R^{(i)}_{\mathcal{A}} \leftarrow [k_{\mathcal{A}}]\, Q_{\mathcal{B}} = [k_{\mathcal{A}} \cdot k_{\mathcal{B}}]\, G$		$R^{(i)}_{\mathcal{B}} \leftarrow [k_{\mathcal{B}}]\, Q_{\mathcal{A}} = [k_{\mathcal{B}} \cdot k_{\mathcal{A}}]\, G$
Use $R^{(i)}_{\mathcal{A}}$		Use $R^{(i)}_{\mathcal{B}}$

Notes:

- A high-level overview of how the above relates to OpenSSL can be found at

  https://wiki.openssl.org/index.php/Diffie_Hellman

  and

  https://wiki.openssl.org/index.php/Elliptic_Curve_Diffie_Hellman

  Note that the former explicitly warns against use of anonymous variants, offering a way to exclude them from the cipher suite list.
- It seems reasonable to say that the static-static and ephemeral-static options are confusion with respect to, e.g., the `ECDHE` cipher suite identifier (which implies ephemeral, but not which, if any party respects this).

Algorithm (EC-DH(E) key agreement [7, Section 8.1][8, Section 2.3])

$\mathcal{A}$		$\mathcal{B}$
Knows $G = E(\mathbb{F}_q) = \langle G \rangle$ of order n, $pk_{\mathcal{B}}$, $(pk_{\mathcal{A}})^{\dagger}$, $(sk_{\mathcal{A}})^{\dagger}$		Knows $G = E(\mathbb{F}_q) = \langle G \rangle$ of order n, $(pk_{\mathcal{A}})^{\dagger}$, $pk_{\mathcal{B}}$, $sk_{\mathcal{B}}$
		$k_{\mathcal{B}} \xleftarrow{\$} \{1, 2, \ldots, n-1\}$ $Q_{\mathcal{B}} \leftarrow [k_{\mathcal{B}}]\, G$
$k^{(i)}_{\mathcal{A}} \xleftarrow{\$} \{1, 2, \ldots, n-1\}$ $Q^{(i)}_{\mathcal{A}} \leftarrow \left[k^{(i)}_{\mathcal{A}}\right] G$		
	$\xrightarrow{Q^{(i)}_{\mathcal{A}}}$	
	$\xleftarrow{Q_{\mathcal{B}}}$	
$R^{(i)}_{\mathcal{A}} \leftarrow \left[k^{(i)}_{\mathcal{A}}\right] Q_{\mathcal{B}} = \left[k^{(i)}_{\mathcal{A}} \cdot k_{\mathcal{B}}\right] G$		$R^{(i)}_{\mathcal{B}} \leftarrow [k_{\mathcal{B}}]\, Q^{(i)}_{\mathcal{A}} = \left[k_{\mathcal{B}} \cdot k^{(i)}_{\mathcal{A}}\right] G$
Use $R^{(i)}_{\mathcal{A}}$		Use $R^{(i)}_{\mathcal{B}}$

Notes:

- A high-level overview of how the above relates to OpenSSL can be found at

  https://wiki.openssl.org/index.php/Diffie_Hellman

  and

  https://wiki.openssl.org/index.php/Elliptic_Curve_Diffie_Hellman

  Note that the former explicitly warns against use of anonymous variants, offering a way to exclude them from the cipher suite list.
- It seems reasonable to say that the static-static and ephemeral-static options are confusion with respect to, e.g., the `ECDHE` cipher suite identifier (which implies ephemeral, but not which, if any party respects this).

A real-world story: an attack [2] on TLS 1.2 + OpenSSL 0.9.8g (5)

Issue 2: (opt-out) ephemeral-static EC-DHE

- Observation(s):
  - good: the key agreement is more efficient (for the server).
  - good: input points are validated by testing whether

    $$P_y^2 \stackrel{?}{=} P_x^3 + a_4 P_x + a_6$$

    given $P = (P_x, P_y)$.
  - bad: ephemeral-*static* EC-DHE is the default i.e.,
    - uses a per-invocation (of the library) rather than a per-session key, *unless*
    - one explicitly uses `SSL_CTX_set_options` using `SSL_OP_SINGLE_ECDH_USE`

    which means $k_{\mathcal{B}}$ is a static, fixed target for any attack.
  - bad: if we select $P = (P_x, P_y)$ as follows
    1. Select P_x such that during the computation of the RHS $t' = (P_x^2 + a_4) \cdot P_x + a_6 \pmod{p}$
       - the step $t'_0 = P_x^2 \pmod{p}$ *does not* trigger the bug, and
       - the step $t'_1 = (t'_0 + a_4) \cdot P_x \pmod{p}$ *does* trigger the bug, and
       - t' is a quadratic residue modulo p.
    2. Compute $P_y = \sqrt{t'} \pmod{p}$.

    then P passes validation, but is on some curve E' rather than E.

Notes:

A real-world story: an attack [2] on TLS 1.2 + OpenSSL 0.9.8g (6)

An attack!

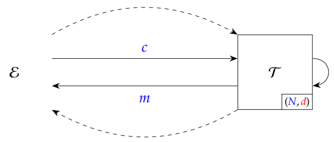
Quote

Decrypting ciphertexts on any computer which multiplies even one pair of numbers incorrectly can lead to full leakage of the secret key, sometimes with a single well-chosen ciphertext.

– *Biham et. al.* [1, *Page 1*]

Notes:

- Scenario:
  - given the following interaction between an **attacker** $\mathcal{E}$ and a **target** $\mathcal{T}$



$m = \text{RSA.Dec}((N, d), c)$
$= c^d \pmod N$

```
1  algorithm RSA.Dec((N, d), c) begin
2    Pre-compute Π = (N, ρ, ω) from N
3    ĉ ← MontMul(Π, c, ρ^2 mod N)
4    r̂ ← MontMul(Π, c, ρ^2 mod N)
5    for i = |d| − 2 downto 0 do
6      r̂ ← MontMul(Π, r̂, r̂)
7      if d_i = 1 then
8        r̂ ← MontMul(Π, r̂, ĉ)
9      end
10   end
11   return MontMul(Π, r̂, 1)
12 end
```

  - and noting that
    - there are no countermeasures implemented,
    - the Montgomery multiplication implementation is FIOS-based [3],
    - the $(w \times w)$-bit integer multiplier hardware has a bug: when computing $r = x \times y$ if

$$\begin{array}{lllll} x \neq \alpha & \vee & y \neq \beta & \Rightarrow & r \text{ is correct} \\ x = \alpha & \wedge & y = \beta & \Rightarrow & r \text{ is } \textit{incorrect} \end{array}$$

    for some known (but arbitrary) α and β.
  - how can $\mathcal{E}$ mount a successful attack, i.e., recover d ?

Notes:

- Attack [1, Section 4.2]:
  - in some t-th step, $\mathcal{E}$
    - knows some more-significant portion of the binary expansion of d, and
    - aims to recover d_t, the next less-significant unknown bit,
  - select a c so during decryption when $i = t$ and just after line #6

$$\begin{array}{lll} \exists j & \text{such that} & \hat{r}_j = \alpha \\ \exists j & \text{such that} & \hat{c}_j = \beta \end{array}$$

    i.e., α and β occur in the representations of $\hat{r}$ and $\hat{c}$,
  - this selection means

$$\begin{array}{lllll} d_t = 0 & \Rightarrow & \hat{r} \text{ is not multiplied by } \hat{c} & \Rightarrow & \text{the bug is not triggered} \\ d_t = 1 & \Rightarrow & \hat{r} \text{ is multiplied by } \hat{c} & \Rightarrow & \text{the bug is triggered} \end{array}$$

  - test whether

$$m^e \pmod N \stackrel{?}{=} c$$

    and infer

$$\begin{array}{lllll} m \text{ is correct} & \Rightarrow & \text{the bug was not triggered} & \Rightarrow & d_t = 0 \\ m \text{ is incorrect} & \Rightarrow & \text{the bug was triggered} & \Rightarrow & d_t = 1 \end{array}$$

Notes:

Feature	Biham et. al. [1, Section 4.2]	Brumley et. al. [2, Section 3]
Target	Fixed d	Fixed $k_{\mathcal{T}}$
Input	Arbitrary poisoned integer $c \in \mathbb{Z}_N^*$	Controlled distinguisher point $Q_{\mathcal{E}} = [k_{\mathcal{E}}]\,G \in E(\mathbb{F}_p)$
Computation	Left-to-right binary exponentiation	Left-to-right (modified) wNAF scalar multiplication
Leakage	Re-encrypt m using e, check against c	Handshake success/failure

Notes:

- Epilogue:
  - good(ish):

Quote

We appreciate you reporting this issue to us but, unfortunately, we aren't inclined to handle this vulnerability because it is already patched and only affects obsolete Linux distributions.

– CERT

https://jscholarship.library.jhu.edu/items/00b58834-a88c-449e-ab23-db2f44207383

Notes:

- The analysis paper by Martin et al. [4] was published in 2013: the attack paper by Brumley et al. [2] was published in 2012, but OpenSSL 0.9.8g was released in2007 (i.e., much earlier).

A real-world story: an attack [2] on TLS 1.2 + OpenSSL 0.9.8*g* (10)

A patch?

- **Epilogue**:
  - bad: even circa 2013, the reality [4] seemed to differ somewhat:

Version	Percentage
0.9.8e-fips-rhel5	37.25
0.9.8g	14.50
0.9.7a	7.02
0.9.8o	4.76
1.0.0-fips	4.36
0.9.7d	2.91
0.9.8n	2.75
0.9.7e	1.94
0.9.8c	1.80
0.9.8m	1.74
0.9.8e	1.72
0.9.8r	1.71

Table 2: Most popular OpenSSL versions on the Internet.

Distribution	OSSL Version	CVEs
Debian Squeeze (6.0)	0.9.8o	11
Debian Lenny (5.0)	0.9.8g	24
Debian Etch (4.0)	0.9.8c	26
RHEL 6	0.9.8e/1.0.0-fips	0/14
RHEL 5	0.9.7a/0.9.8e-fips	14/0
RHEL 4	0.9.6b/0.9.7a	9/14
Fedora 18	1.0.1c	3
Fedora 17	1.0.0i	3
Fedora 16	1.0.0e	9

Table 3: Default OpenSSL versions shipping with popular Linux distributions.

https://jscholarship.library.jhu.edu/items/00b58834-a88c-449e-ab23-db2f44207383

Notes:

- The analysis paper by Martin et al. [4] was published in 2013: the attack paper by Brumley et al. [2] was published in 2012, but OpenSSL 0.9.8*g* was released in2007 (i.e., much earlier).

Unit summary (1)

- **Summary**:

https://memegenerator.net

Notes:

Unit summary (2)

- **Summary**: what *have* we done includes
  1. focused on some high-level outcomes:
     - improved

$$\left.\begin{array}{c}\text{awareness}\\ \text{understanding}\\ \text{skills}\\ \vdots\end{array}\right\} \Rightarrow \text{ ability to engage with problems, produce solutions, ...}$$

     - general concepts (versus specific examples) $\Rightarrow$ long-term (versus short-term) value.
  2. highlighted some high-level principles:
     - most effective implementation will be domain-specific,
     - apply adversarial thinking to *everything*,
     - need for and value in well-considered trade-offs,
     - don't *over*-optimise to the point efficiency > security,
     - apply "*inverse* Postel's Law", i.e., be *very* strict re. what you accept as input,
     - ...
  3. exposed some low-level detail:
     - tools, techniques, and technologies,
     - shift from abstract toward and including concrete (e.g., AES versus generic block cipher),
     - written standards, RFCs, etc. (e.g., FIPS-197 versus lecture slides),
     - ...

Notes:

Unit summary (3)

- **Summary**: what *haven't* we done includes
  1. greater *depth*, i.e., more X for $X \in$ COMS30048:
     - more implementation
       - platforms (e.g., FPGAs, ASICs, GPUs, ..., JavaScript versus C)
       - constraints (e.g., from use-case, platform, tooling, ...)
       - co-design (e.g., hardware/software, specification/implementation, ...)
       - ...
     - more attacks
     - more countermeasures
     - more primitives (e.g., PQC, LWC, hash functions, ..., FHE, MPC, ...)
     - more protocols (e.g., DNSSEC, IPSec, ...)
  2. greater *breadth*, i.e., more X for $X \notin$ COMS30048:
     - hardware security (e.g., TEEs, HSMs, secure boot and update, FDE, ...)
     - formal verification
     - key management (e.g., secure generation, storage, and erasure, ...)
     - social-technical (e.g., usability, politics, risk analysis, supply chain, disclosure, ...)
     - certification and standardisation processes
     - ...

Notes:

References

[1] E. Biham, Y. Carmeli, and A. Shamir. "Bug Attacks". In: *Advances in Cryptology (CRYPTO)*. Vol. 5157. LNCS. Springer-Verlag, 2008, pp. 221–240 (see pp. 23, 27, 29).

[2] B. Brumley et al. "Practical realisation and elimination of an ECC-related software bug attack". In: *Topics in Cryptology (CT-RSA)*. LNCS 7178. Springer-Verlag, 2012, pp. 171–186 (see pp. 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31–34).

[3] Ç.K. Koç, T. Acar, and B.S. Kaliski. "Analyzing and comparing Montgomery multiplication algorithms". In: *IEEE Micro* 16.3 (1996), pp. 26–33 (see p. 25).

[4] P.D. Martin et al. *Classifying Network Protocol Implementation Versions: An OpenSSL Case Study*. Tech. rep. 13-01. Johns Hopkins University, 2013. URL: `http://www.michaelrushanan.org/pdf/martin.pdf` (see pp. 31–34).

[5] H. Reimann. `BN_nist_mod_384` *gives wrong answers*. `openssl-dev` mailing list #1593. 2007. URL: `http://marc.info/?t=119271238800004` (see p. 7).

[6] J.A. Solinas. *Generalized Mersenne Numbers*. Tech. rep. CORR 99-39. Centre for Applied Cryptographic Research (CACR), University of Waterloo, 1999 (see p. 9).

[7] T. Dierks and E. Rescorla. *The Transport Layer Security (TLS) Protocol version* 1.2. Internet Engineering Task Force (IETF) Request for Comments (RFC) 5246. 2008. URL: `http://tools.ietf.org/html/rfc5246` (see pp. 15, 17, 19).

[8] E. Rescorla. *Diffie-Hellman Key Agreement Method*. Internet Engineering Task Force (IETF) Request for Comments (RFC) 2631. 1999. URL: `http://tools.ietf.org/html/rfc2631` (see pp. 15, 17, 19).

Notes: